

Lattice calculation of the vector spectral functions and thermal dilepton rate

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Based on work with

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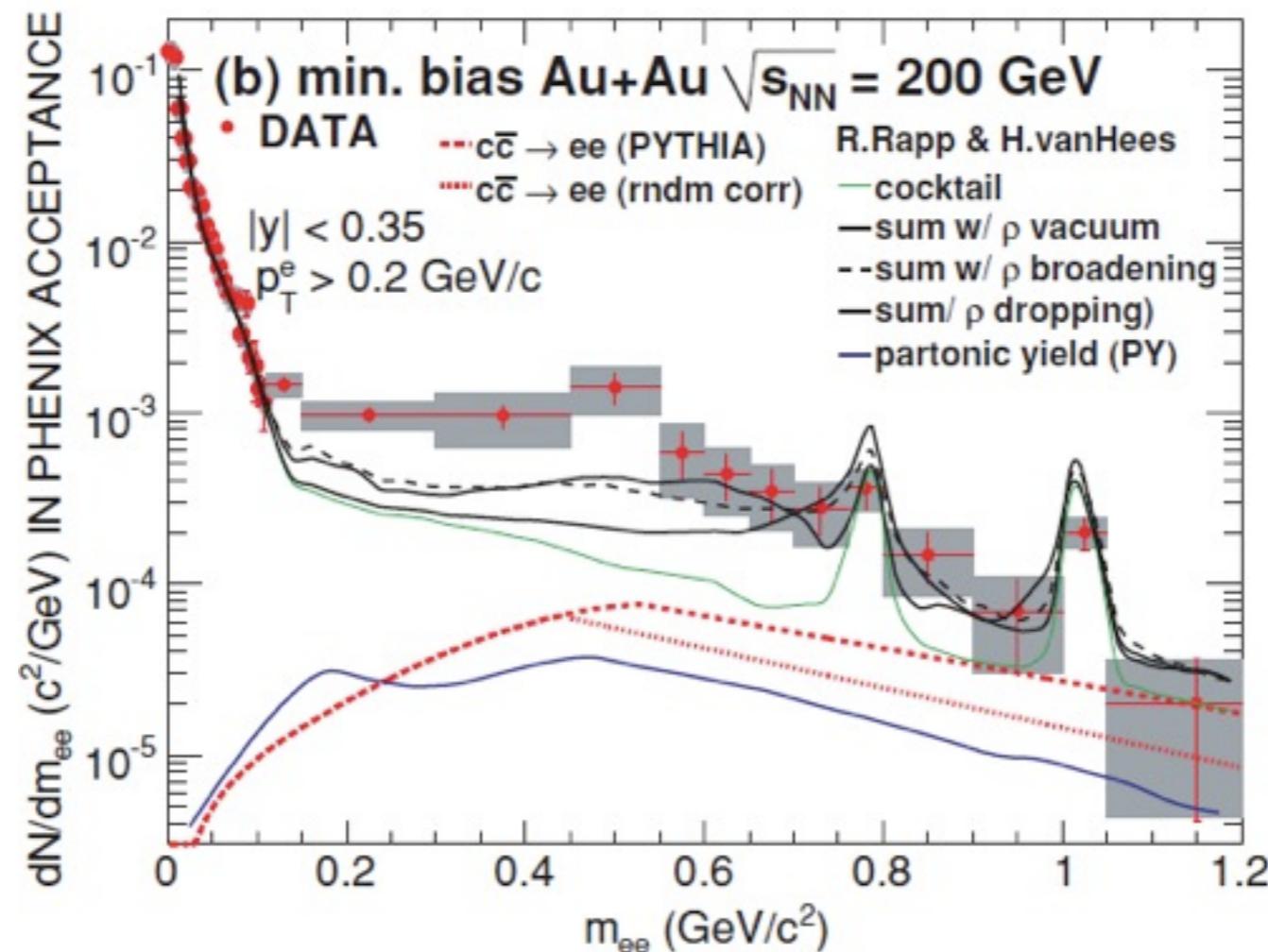
Phys.Rev.D83:034504,2011, arXiv:1012.4963,
J.Phys.G38:124178,2011, arXiv:1109.4054



Outline

- **Introduction & Motivation**
 - thermal dilepton & photon emission rate, electrical conductivity
 - Euclidean correlation and spectral functions
- **Vector correlation functions on the lattice**
 - finite volume & cut-off effects
 - thermal moments of spectral functions
 - continuum extrapolation
- **Thermal dilepton rates and electric conductivity**
 - chi-square fitting with Ansätz
 - Maximum Entropy Method analysis without Ansätz
- **Conclusions & Outlook**

Motivation: PHENIX results on dilepton rates

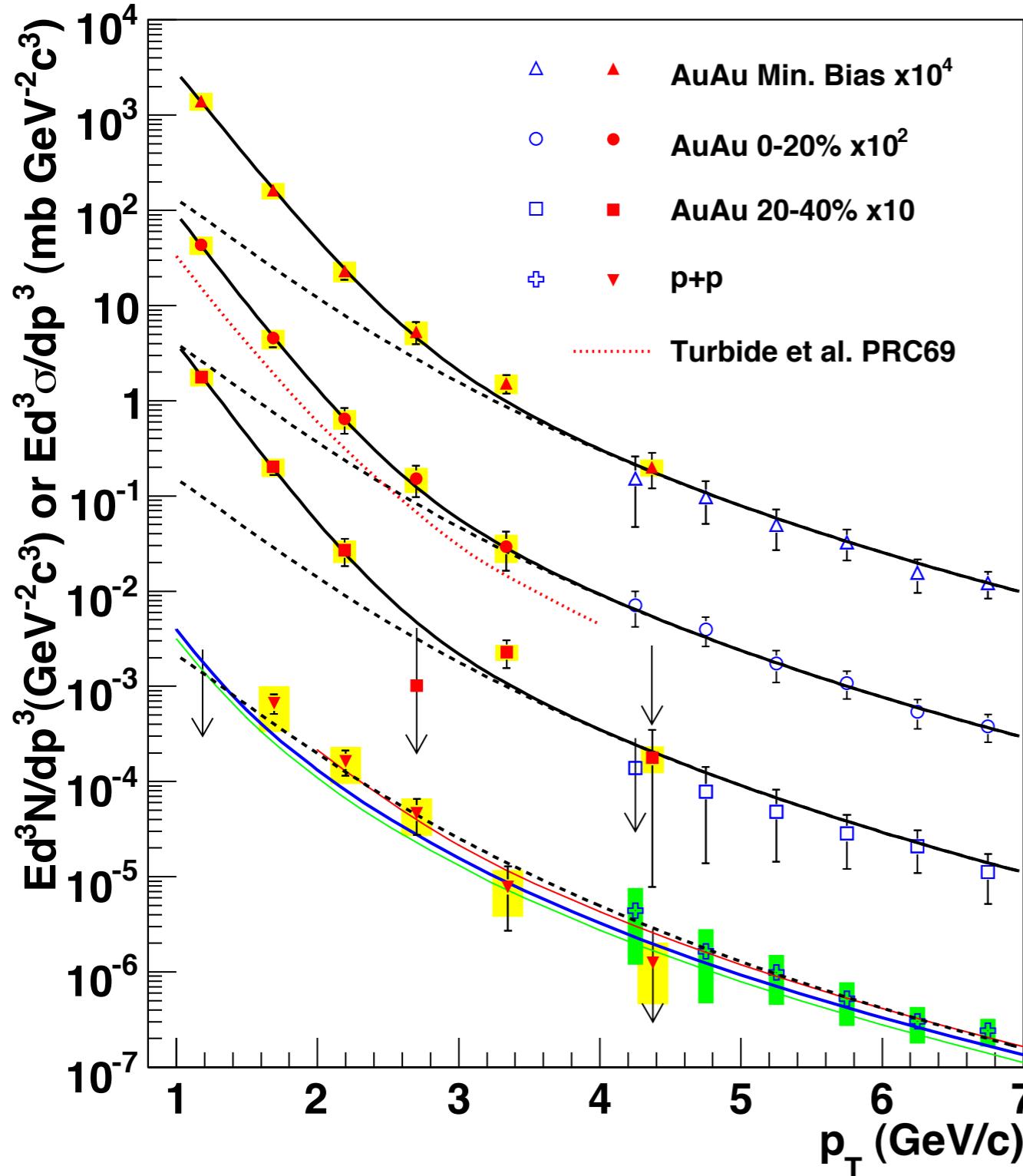


- the low mass, the low pt PHENIX puzzle (c.f. Bratkovskaya et al, Dusling et al)

- dilepton rates:

$$\frac{dN_{l^+l^-}}{d\omega d^3p} = C_{em} \frac{\alpha_{em}^2}{6\pi^3} \frac{\rho_V(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)} \quad C_{em} = e^2 \sum_{f=1}^{n_f} Q_f^2$$

Direct photon production



- pp data well described by NLO pQCD at $p_T > 2 \text{ GeV}/c$
- Enhanced direct photon production in AuAu at $p_T < 2.5 \text{ GeV}/c$
- photon emission rate from a thermal medium:

$$\omega \frac{dR_\gamma}{d^3p} = C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho_V(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1}$$

Vector correlation & spectral functions

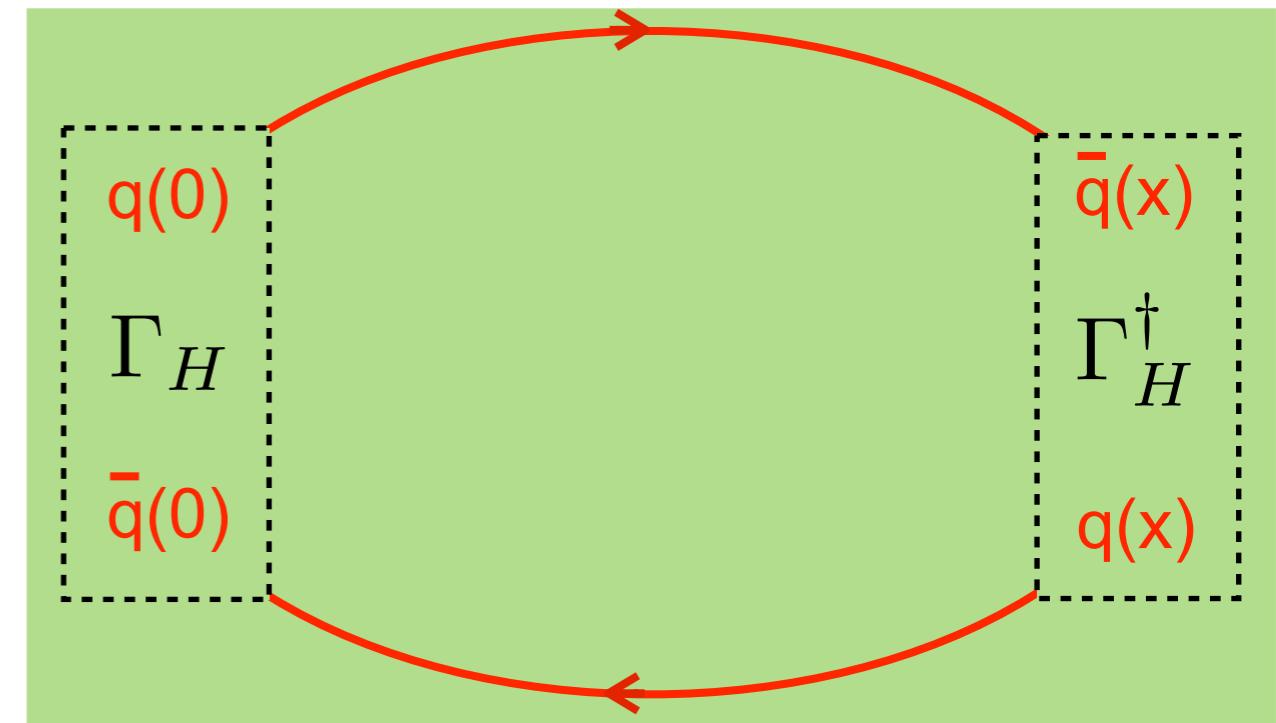
Spectral function

$$\rho(\omega, \vec{p}) = D^+(\omega, \vec{p}) - D^-(\omega, \vec{p}) = 2 \operatorname{Im} D_R(\omega, \vec{p})$$

Euclidean correlation function

$$G_{\mu\nu}(\tau, \vec{p}) = \int d^3x \langle J_\mu(\tau, \vec{x}) J_\nu^\dagger(0, \vec{0}) \rangle e^{i\vec{p}\cdot\vec{x}}$$

$$J_\mu(\tau, \vec{x}) \equiv \bar{q}(\tau, \vec{x}) \gamma_\mu q(\tau, \vec{x})$$



Spectral representation

$$G(\tau, \vec{p}) = \int d^3x e^{-i\vec{p}\cdot\vec{x}} D^+(-i\tau, \vec{x}), \quad D^+(t, \vec{x}) = D^-(t + i\beta, \vec{x})$$

$$G_H(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}, T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} , \quad H = 00, ii, V .$$

Vector correlation function

$$G_H(\tau, \vec{p}, T) = \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega, \vec{p}, T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} , \quad H = 00, ii, V .$$

p=0 in this work

time like correlator G_{00} and space like correlator G_{ii}

$$G_V(\tau, \vec{p}, T) = G_{ii}(\tau, \vec{p}, T) + G_{00}(\tau, \vec{p}, T)$$

conserved current, J_0 , gives τ -independent correlator G_{00}

$$G_{00}(T) \equiv -\chi_q T + \mathcal{O}(a^2)$$

the local, non-conserved current needs to be renormalized

$$J_\mu(\tau, \vec{x}) = 2\kappa Z_V \bar{\psi}(\tau, \vec{x}) \gamma_\mu \psi(\tau, \vec{x})$$

avoid ambiguities of renormalization

$$R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau)} ; \quad R(\tau) \equiv \frac{G_V(\tau)}{G_{00}(\tau) G_V^{free}(\tau T)}$$

Prior information on spectral functions

- free vector spectral function (in the infinite temperature limit)

$$\rho_{00}^{free}(\omega) = -2\pi T^2 \omega \delta(\omega)$$

$$\rho_{ii}^{free}(\omega) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi} \omega^2 \tanh\left(\frac{\omega}{4T}\right)$$

- ◆ δ -functions cancel in $\rho_V(\omega) \equiv \rho_{00}(\omega) + \rho_{ii}(\omega)$

- vector spectral function at $T < \infty$

- ◆ δ -function in ρ_{00} is protected

$$\rho_{00}(\omega, T) = -2\pi \chi_q \omega \delta(\omega)$$

- ◆ δ -function in ρ_{ii} is smeared out

possible form: Breit-Wigner (BW) form + modified continuum

$$\rho_{ii}(\omega, T) = \cancel{\chi_q} \cancel{c_{BW}} \frac{\omega \Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} \left(1 + \frac{\alpha_s}{\pi}\right) \omega^2 \tanh\left(\frac{\omega}{4T}\right)$$

3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \alpha_s$

previous lattice results on electrical conductivity

Electrical conductivity:

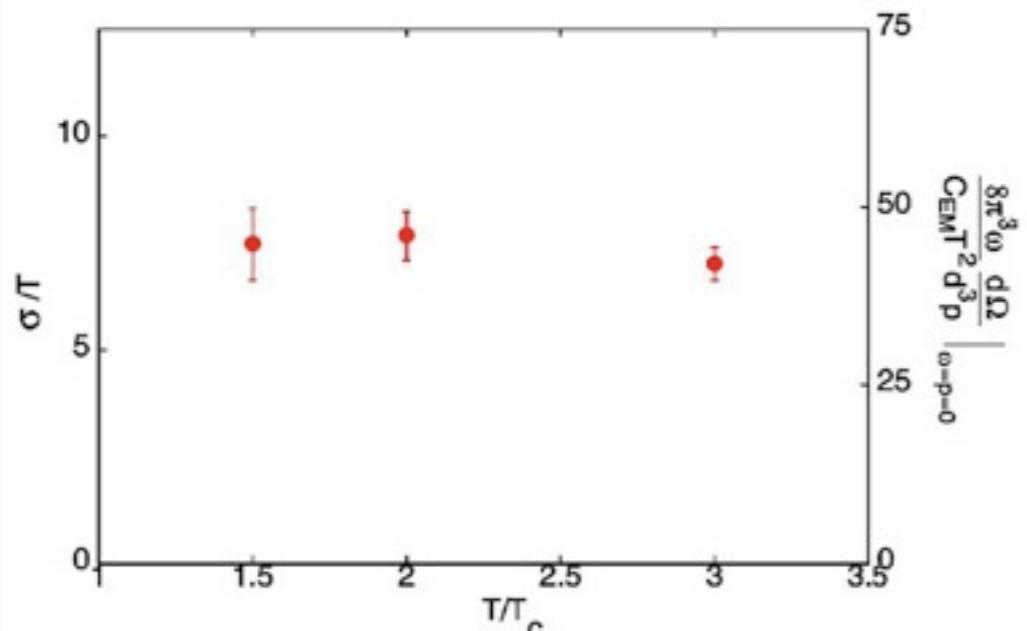
$$\frac{\sigma}{T} = \frac{C_{em}}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega)}{\omega T}$$

The emission rate of soft photons:

$$\lim_{\omega \rightarrow 0} \omega \frac{dR_\gamma}{d^3p} = \lim_{\omega \rightarrow 0} C_{em} \frac{\alpha_{em}}{4\pi^2} \frac{\rho(\omega = |\vec{p}|, T)}{e^{\omega/T} - 1} = \frac{3}{2\pi^2} \sigma(T) T \alpha_{em}$$

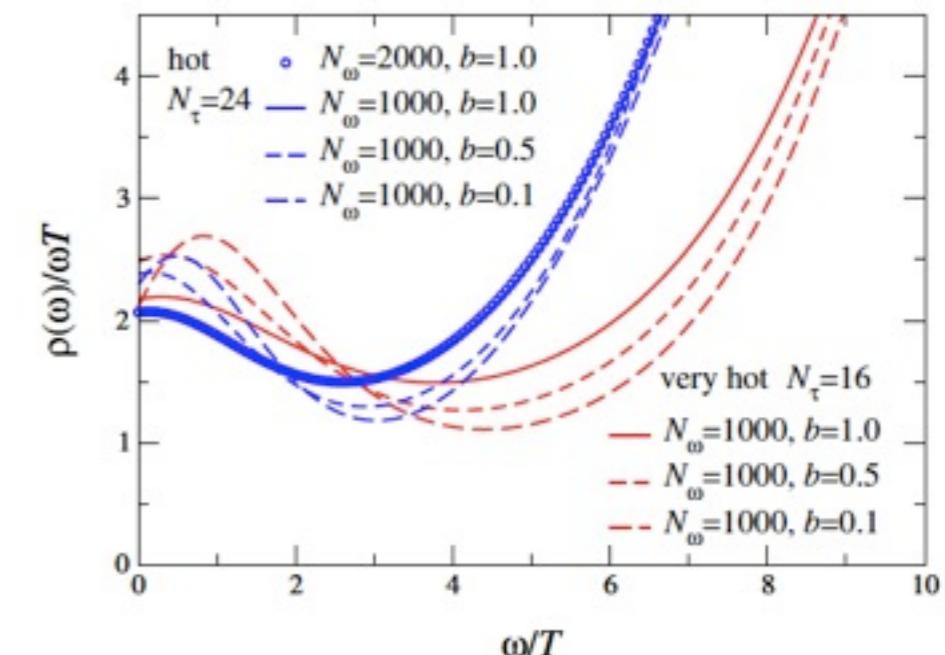
Quite different results from previous lattice calculations:

$$\sigma/T \simeq 7C_{em}$$



S. Gupta PLB 597(2004)57

$$\sigma/T \simeq (0.4 \pm 0.1)C_{em}$$



G.Aarts et al., PRL 99(2007)

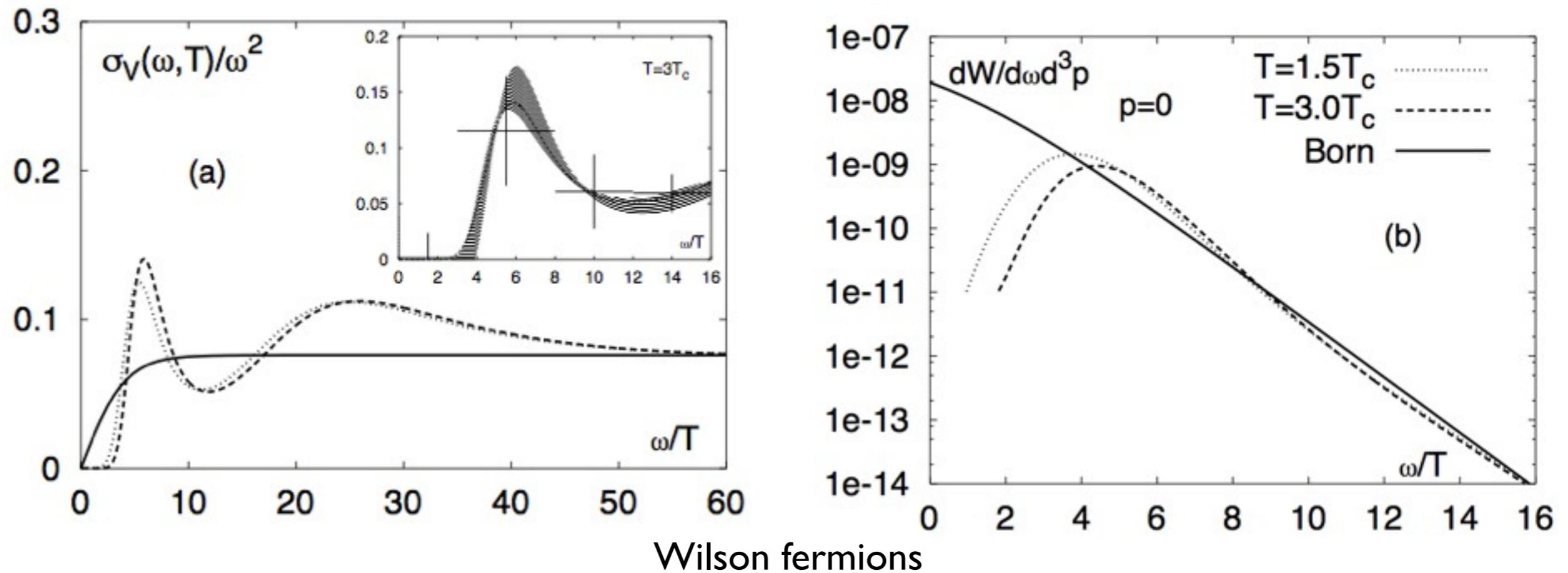
$N_\tau = 8 - 14, N_\sigma \leq 44$

staggered fermions used

ρ_{even} and ρ_{odd} need to be distinguished

$N_\tau = 16, 24, N_\sigma = 64$

previous lattice results on thermal dilepton rate



F. Karsch, E. Laermann, P. Petreczky, S. Stickan and I. Wetzorke, Phys.Lett. B530 (2002) 147-152

- $64^3 \times 16$ lattices, finite volume & lattice cutoff effects?
- N_T not sufficiently large to extract spectral function
- $\rho(\omega)$ should be linear in ω at small ω , not captured by the MEM analysis
- Integral Kernel needs to be redefined in MEM to explore low frequency region

G. Aarts et al, PRL '07

Vector correlation functions on large & fine lattices

- SU(3) gauge configurations at $T/T_c \approx 1.45$
- lattice size $N_\sigma^3 \times N_\tau$ with $N_\sigma = 32-128$ & $N_\tau = 16, 24, 32, 48$
- Non-perturbatively clover O(a) improved Wilson fermions
- Quark masses close to chiral limit $\kappa \simeq \kappa_c$

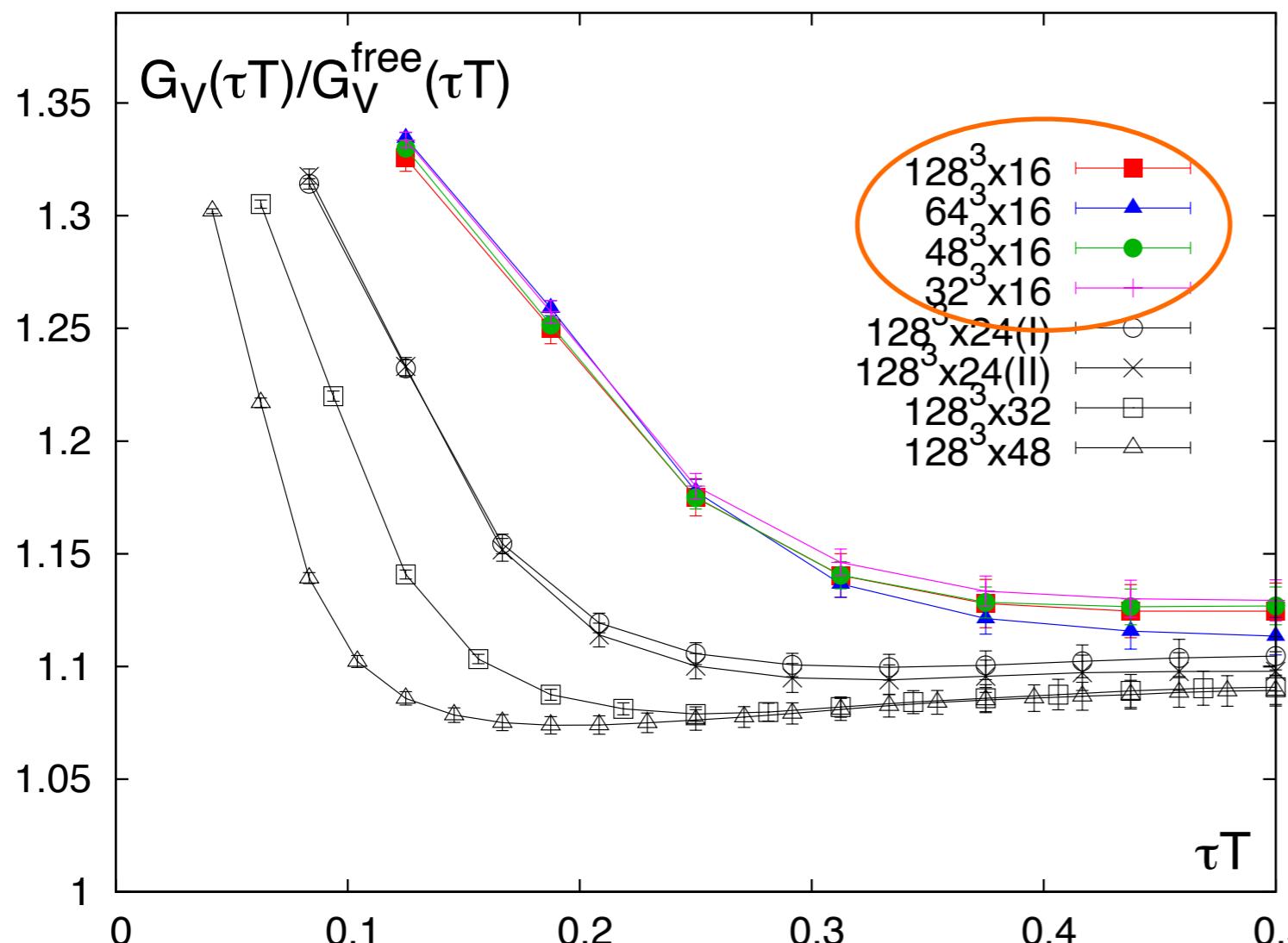
volume dependence

N_τ	N_σ	β	c_{SW}	κ	Z_V	$a^{-1}[\text{GeV}]$	$a[\text{fm}]$	#conf
16	32	6.872	1.4125	0.13495	0.829	6.43	0.031	251
16		6.872	1.4125	0.13495	0.829	6.43	0.031	229
16		6.872	1.4125	0.13495	0.829	6.43	0.031	191
16		6.872	1.4125	0.13495	0.829	6.43	0.031	191
24	128	7.192	1.3673	0.13431	0.842	9.65	0.020	340
		7.192	1.3673	0.13440	0.842	9.65	0.020	156
32	128	7.457	1.3389	0.13390	0.851	12.86	0.015	255
48	128	7.793	1.3104	0.13340	0.861	18.97	0.010	451

cut-off dep.
& continuum
extrapolation

close to continuum

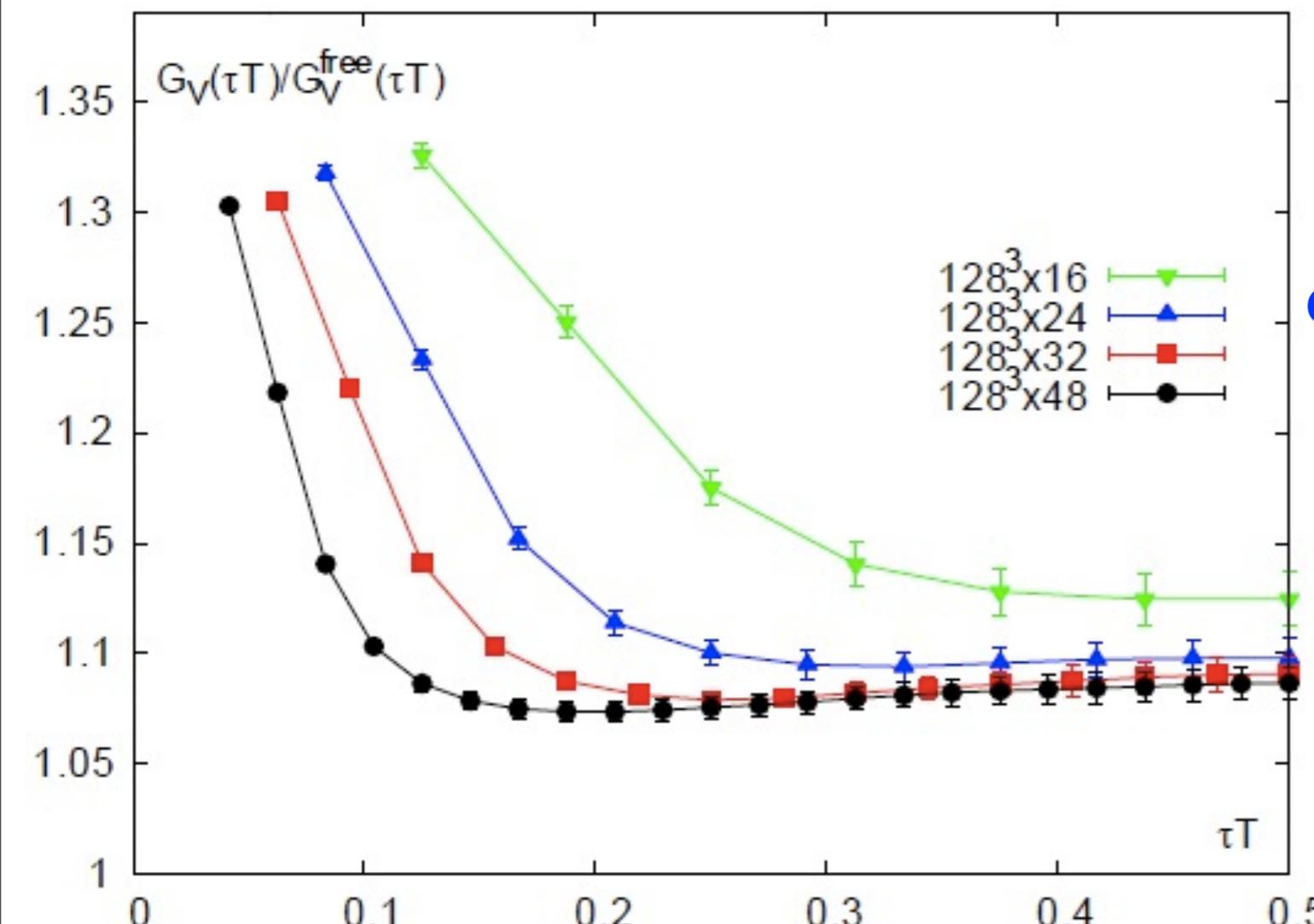
Volume & cut-off dep. of vector corr. function



Normalized by free correlators in the continuum $G_V^{\text{free}}(\tau T)$

$$G_V^{\text{free}}(\tau T) = 6T^3 \left(\pi (1 - 2\tau T) \frac{1 + \cos^2(2\pi\tau T)}{\sin^3(2\pi\tau T)} + 2 \frac{\cos(2\pi\tau T)}{\sin^2(2\pi\tau T)} \right)$$

Volume & cut-off dep. of vector corr. function



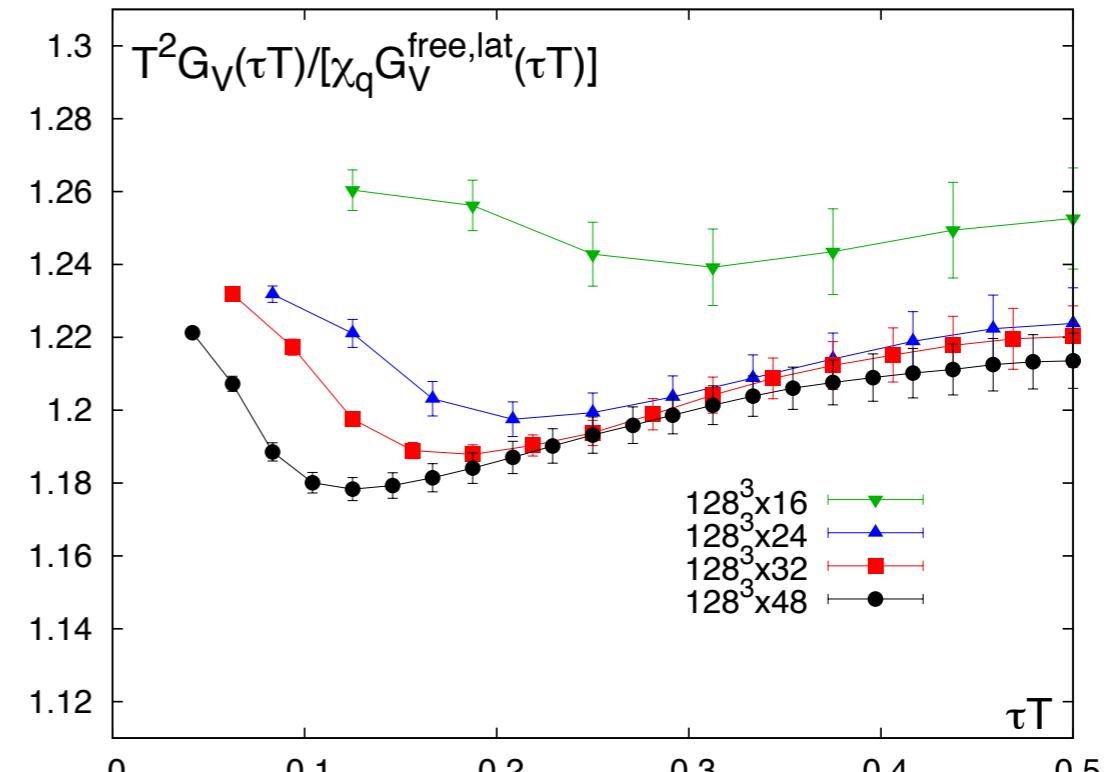
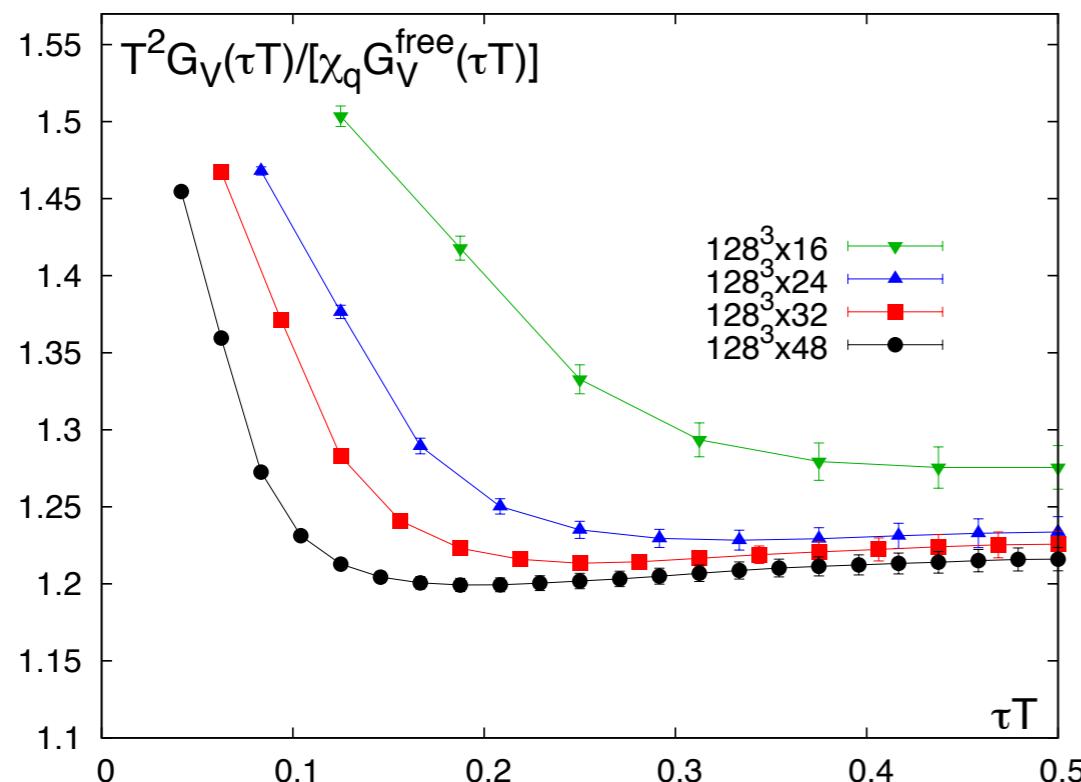
cut-off effects are more sever than finite volume effects

large N_T needed to perform continuum extrapolation

$G_V(\tau T)$ is close to the free case at large τT

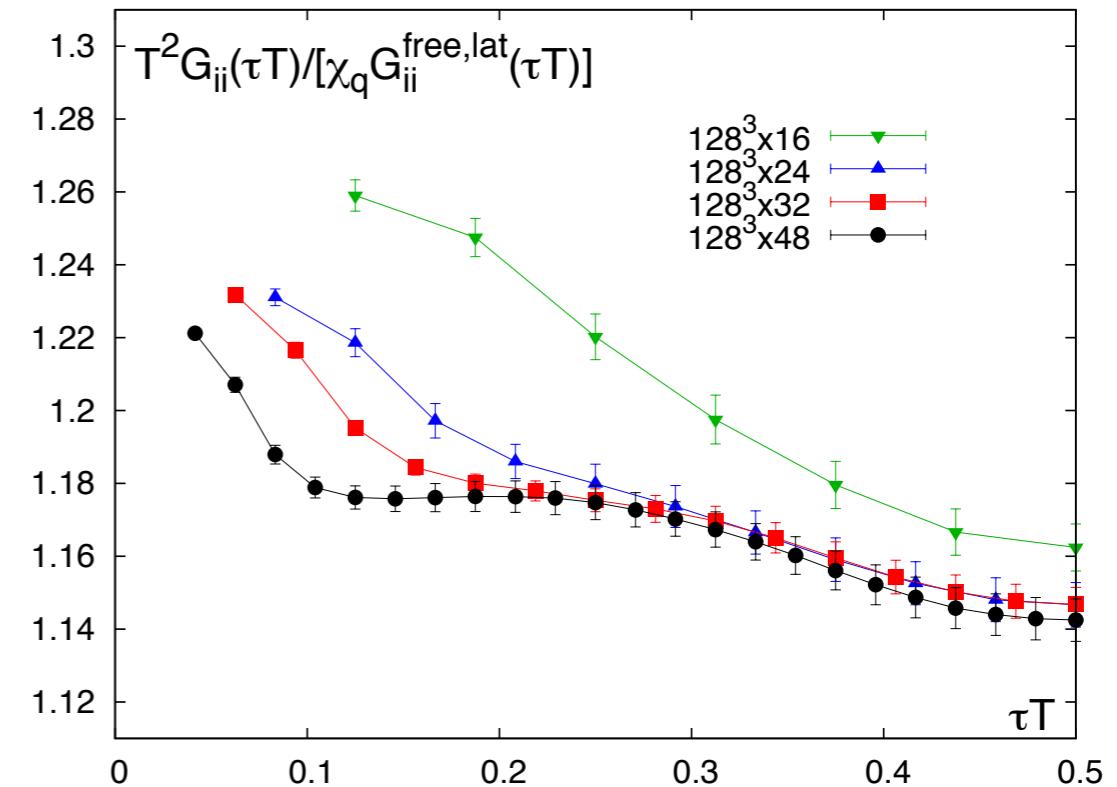
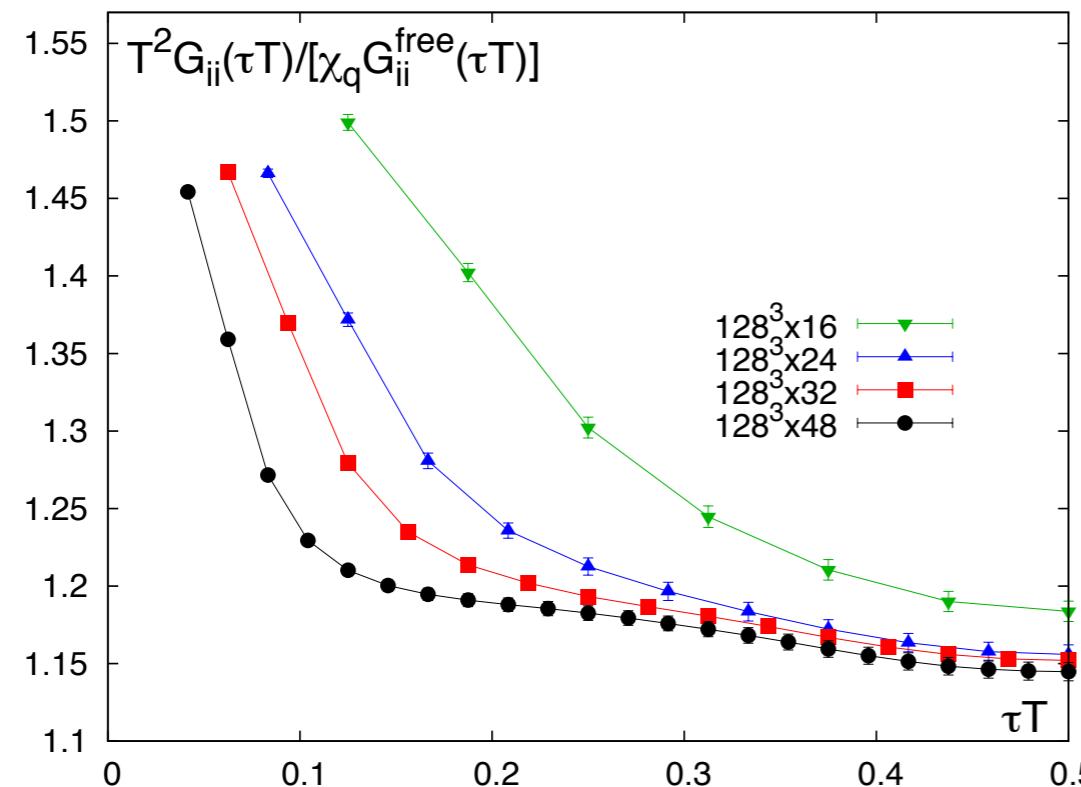
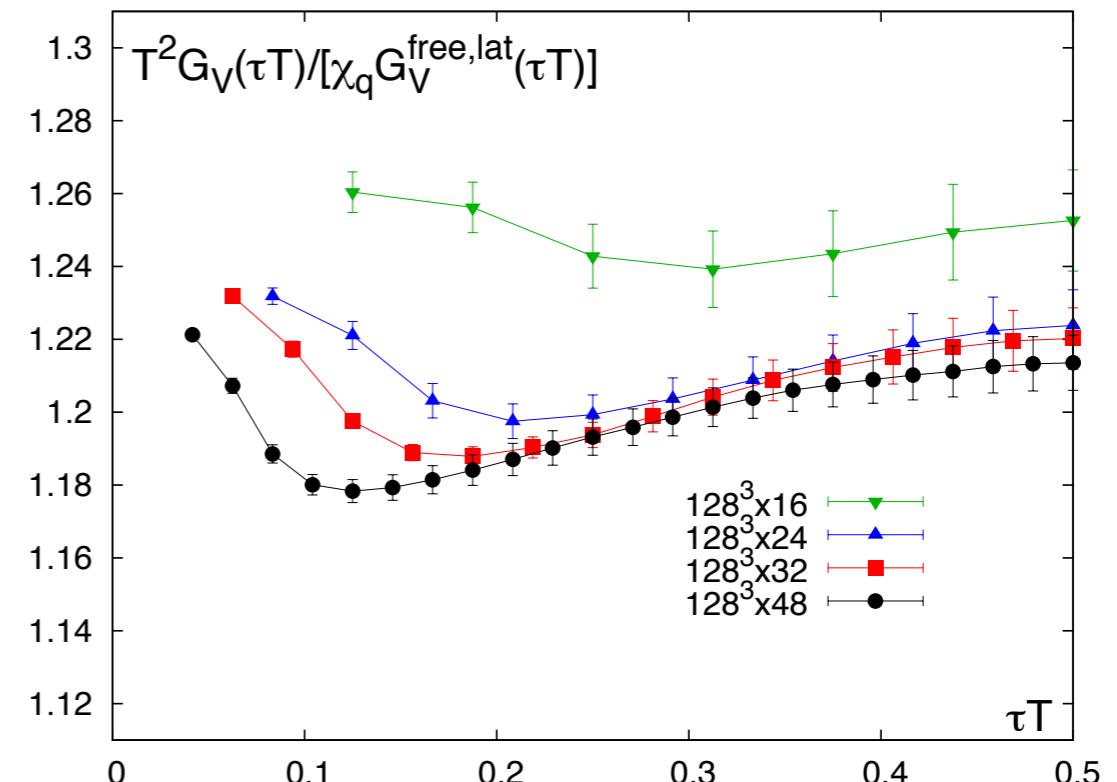
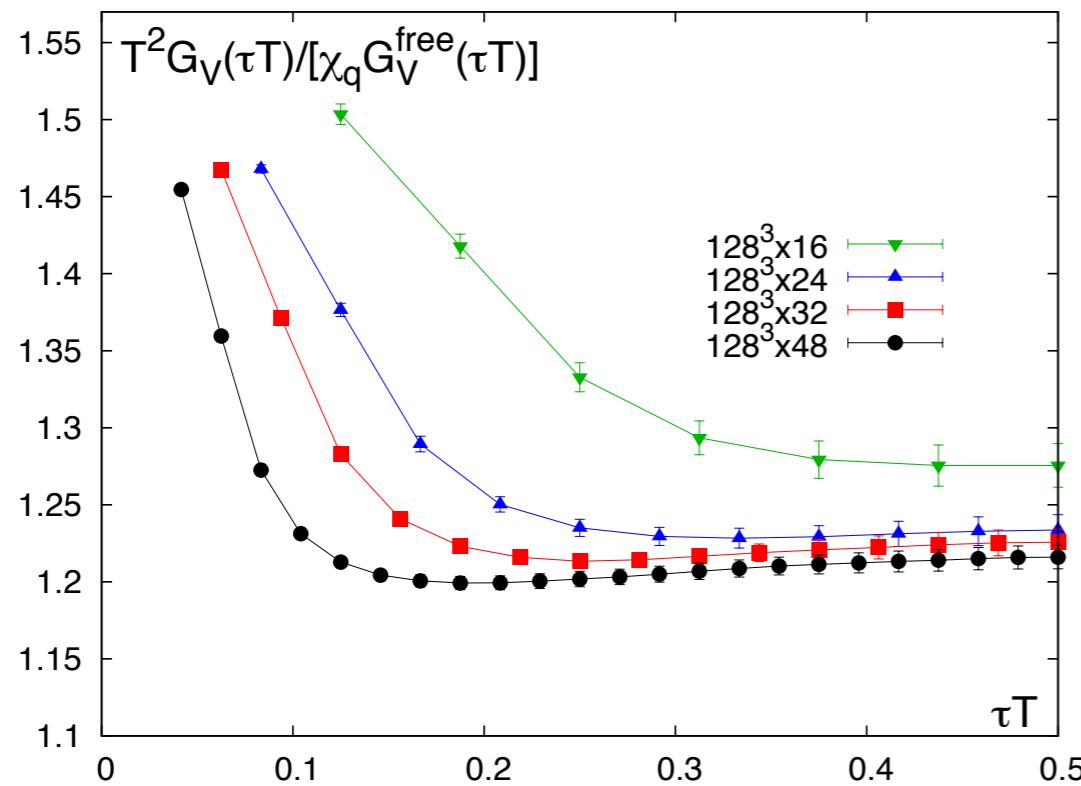
incomplete cancelation between $G_{00}(\tau T)$ and BW-contribution to $G_{ii}(\tau T)$?

Cut-off effects of vector corr. function

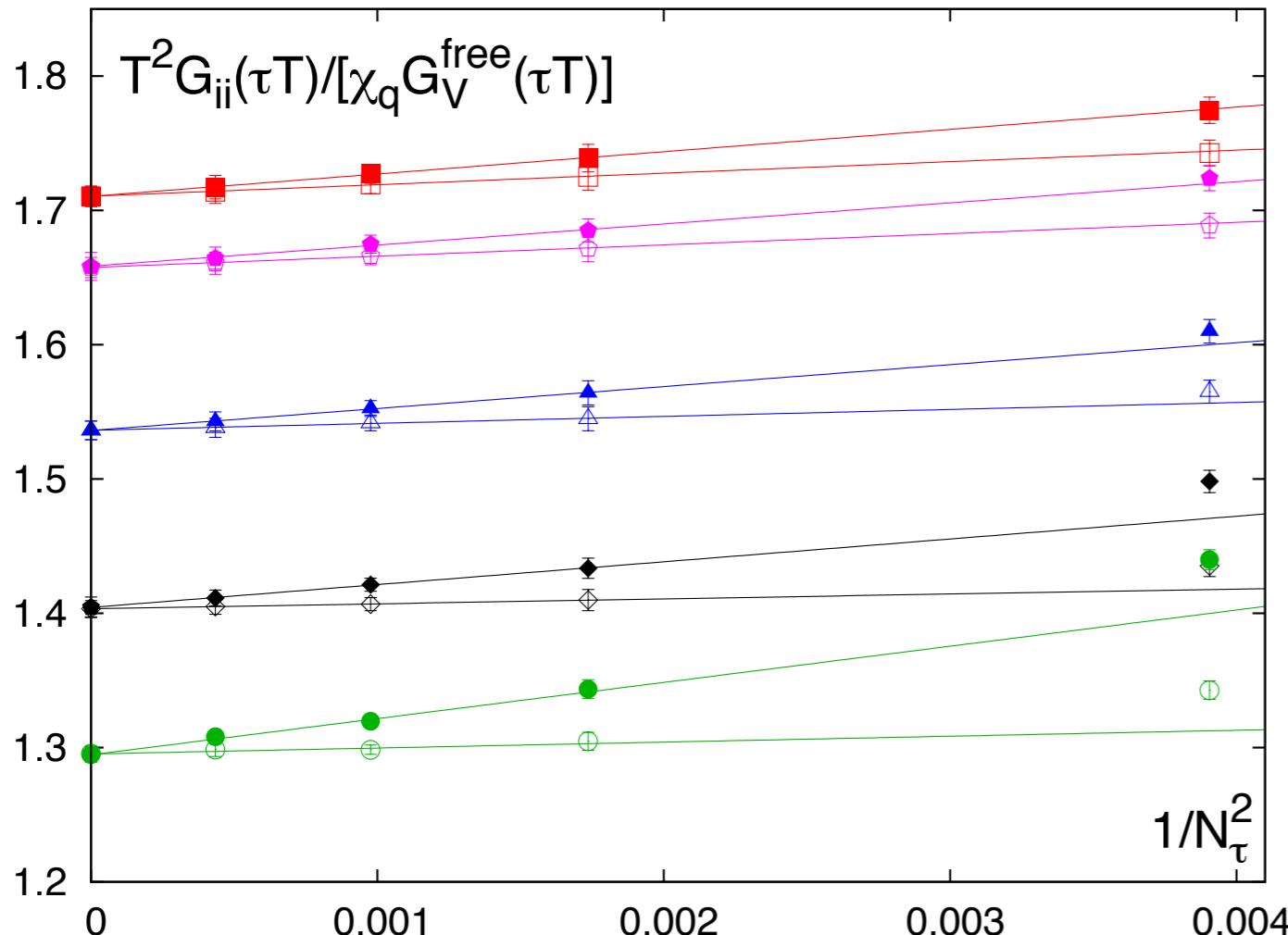


- ratios are free of renormalization ambiguities
- most of the cut-off effects are well described by discretization errors of non-interacting case

Cut-off effects of vector corr. function



Continuum extrapolation

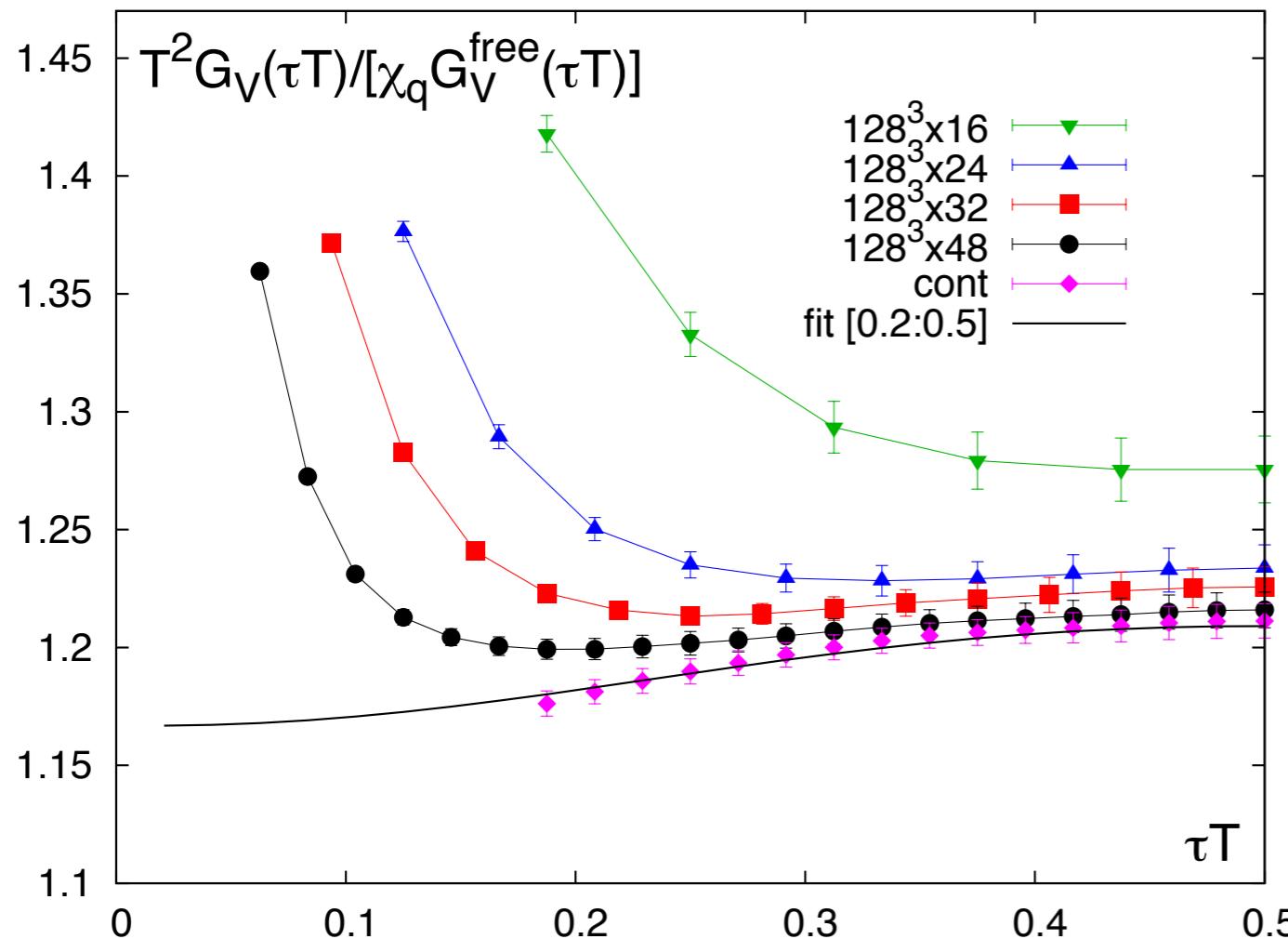


- extrapolation in $(aT)^2 = 1/N_\tau^2$

N_τ	∞
χ_q/T^2	0.897(3)
$G_V^{(2)}/(\tilde{\chi}_q G_V^{(2),free})$	1.189(13)
$G_V(1/2)/(\tilde{\chi}_q G_V^{free}(1/2))$	1.211(9)
$G_V(1/4)/(\tilde{\chi}_q G_V^{free}(1/4))$	1.190(7)
$G_{ii}(1/2)/(\tilde{\chi}_q G_{ii}^{free}(1/2))$	1.142(9)
$G_{ii}(1/4)/(\tilde{\chi}_q G_{ii}^{free}(1/4))$	1.172(7)

- extrapolation at the other values of τT using spline interpolation on data at fixed cut-off
- extrapolation under control for $\tau T \gtrsim 0.2$

Continuum extrapolation



$$\frac{G_V(1/2)}{G_V^{\text{free}}(1/2)} = 1.086 \pm 0.008 ,$$

$$\frac{G_V(1/4)}{G_V^{\text{free}}(1/4)} = (0.982 \pm 0.005) \frac{G_V(1/2)}{G_V^{\text{free}}(1/2)}$$

- Increase of $G_V(\tau T) / G_V^{\text{free}}(\tau T)$ with τT is obvious
- The rise with τT indicates that vector spectral function in the low frequency region is different from the free case
- Motivation for the Breit-Wigner type ansatz fitting

Curvature of vector correlator at $\tau T = 1/2$

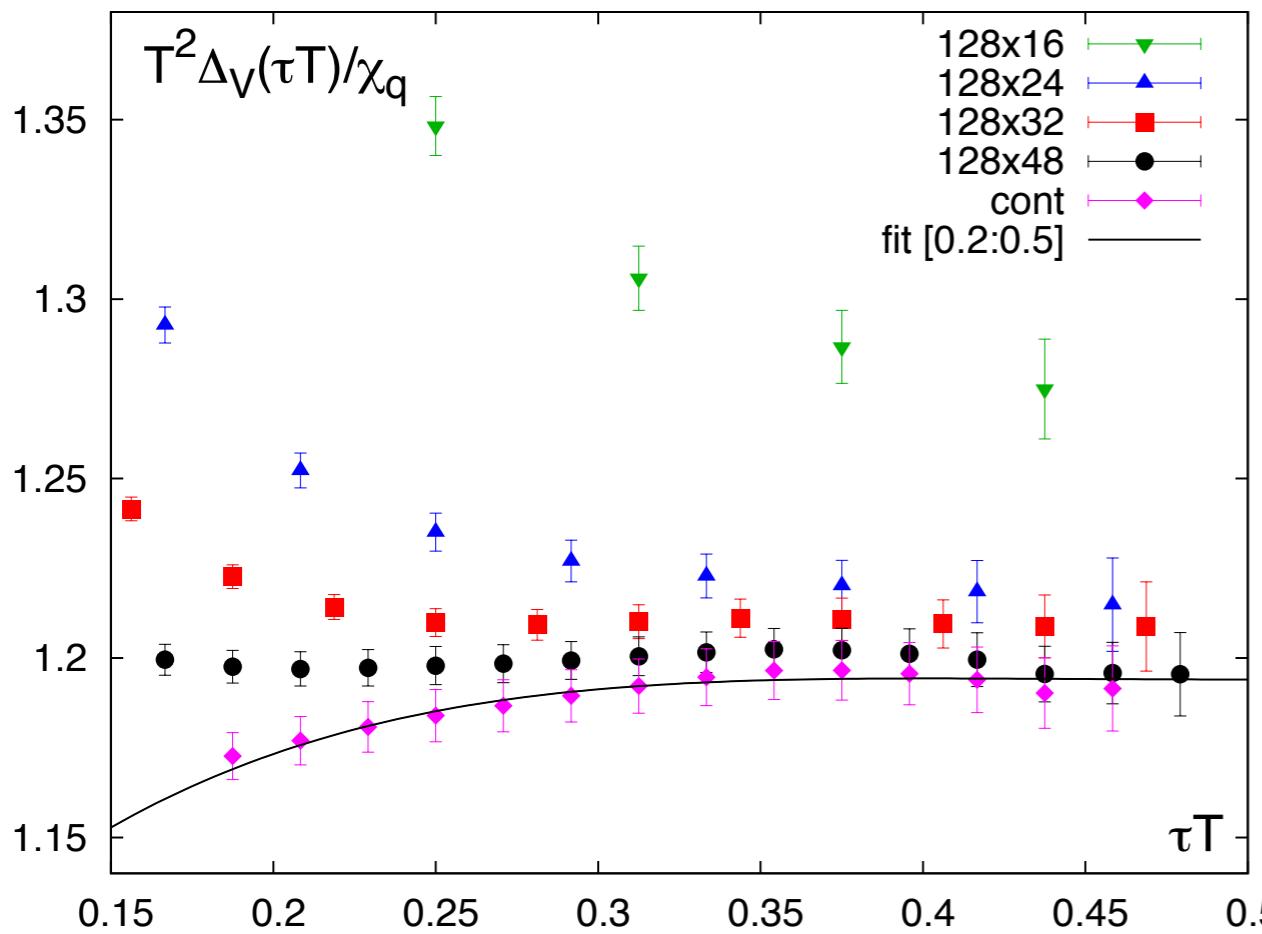
Taylor expansion around the mid-point $\tau T = 1/2$

$$G_V(\tau T) = G_V^{(0)} \sum_{n=0}^{\infty} \frac{G_V^{(2n)}}{G_V^{(0)}} \left(\tau T - \frac{1}{2}\right)^{2n} = \int_0^{\infty} \frac{d\omega}{2\pi} \frac{\rho_V(\omega)}{\sinh(\omega/2T)} \left[1 + \frac{1}{2!} \left(\frac{\omega}{T}\right)^2 (\tau T - \frac{1}{2})^2 + \frac{1}{4!} \left(\frac{\omega}{T}\right)^4 (\tau T - \frac{1}{2})^4 + \dots\right]$$

Thermal moments of vector spectral functions

$$G_V^{(n)} = \frac{1}{n!} \left. \frac{d^n G_V(\tau T, T)}{d(\tau T)^n} \right|_{\tau T = 1/2} = \frac{1}{n!} \int_0^{\infty} \frac{d\omega}{2\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho_V(\omega)}{\sinh(\omega/2T)}, \quad G_V^{(0)} = G_V(\tau T = 1/2)$$

ratio of mid-point subtracted correlation functions



$$\begin{aligned} \Delta_V(\tau T) &\equiv \frac{G_V(\tau T) - G_V^{(0)}}{G_V^{free}(\tau T) - G_V^{(0),free}} \\ &= \frac{G_V^{(2)}}{G_V^{(2),free}} \left(1 + \left(\frac{G_V^{(4)}}{G_V^{(2)}} - \frac{G_V^{(4),free}}{G_V^{(2),free}}\right) \left(\tau T - \frac{1}{2}\right)^2 + \dots\right) \end{aligned}$$

↓

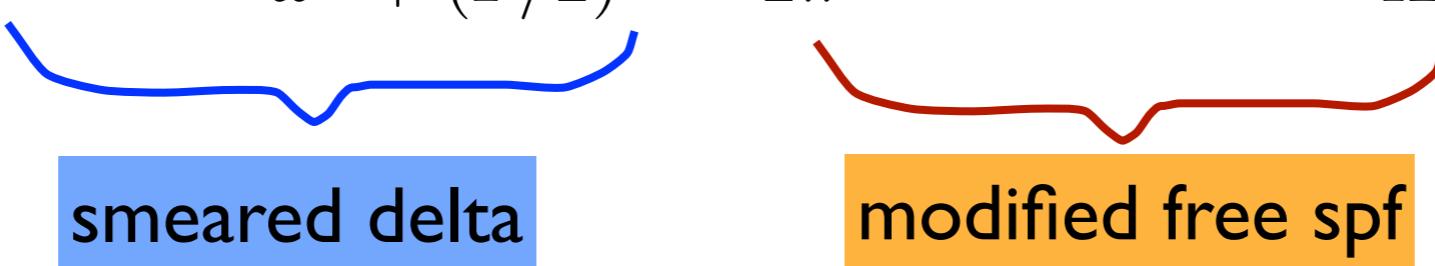
$$\begin{aligned} \frac{G_V^{(2)}}{G_V^{(0)}} &= (0.982 \pm 0.012) \frac{G_V^{(2),free}}{G_V^{(0),free}}, \\ \frac{G_{ii}^{(2)}}{G_{ii}^{(0)}} &= (1.043 \pm 0.010) \frac{G_{ii}^{(2),free}}{G_{ii}^{(0),free}}. \end{aligned}$$

Breit-Wigner + continuum Ansatz

- The correlation function at $\tau T = 1/2$ is about 2% larger than the corresponding free field value: $G_{ii}(1/2)/G_{ii}^{free}(1/2) = 1.024(8)$
- The second moment of the vector spectral function deviates from the free field value by about 7%: $G_V^{(2)}/G_V^{(2),free} = G_{ii}^{(2)}/G_{ii}^{(2),free} = 1.067(12)$
- The deviation from the free field value increase with increasing Euclidean time: $G_V(1/4)/G_V^{free}(1/4) = (0.982 \pm 0.005)G_V(1/2)/G_V^{free}(1/2)$

Fitting Ansatz

$$\rho_{00}(\omega, T) = -2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega, T) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\omega^2 \tanh\left(\frac{\omega}{4T}\right)$$


smeared delta

modified free spf

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Fitting Ansatz

$$\frac{\sigma}{T} = \frac{1}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega)}{\omega T} = \frac{C_{em}}{3} \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}}$$

$\rho_{ii}(\omega, T) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1+k) \omega^2 \tanh\left(\frac{\omega}{4T}\right)$

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$$\rho_{ii}(\omega, T) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1+k) \omega^2 \tanh\left(\frac{\omega}{4T}\right)$$

$$\tilde{\rho}_{ii}(\tilde{\omega}) = \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \frac{2\tilde{\omega}(\tilde{\Gamma}/2)^2}{\tilde{\omega}^2 + (\tilde{\Gamma}/2)^2} + \frac{3}{2\pi} (1+k) \tilde{\omega}^2 \tanh\left(\frac{\tilde{\omega}}{4}\right)$$

Fit to correlators & thermal moments

$$\tilde{\rho}_{ii}(\tilde{\omega}) = \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \frac{2\tilde{\omega}(\tilde{\Gamma}/2)^2}{\tilde{\omega}^2 + (\tilde{\Gamma}/2)^2} + \frac{3}{2\pi} (1+k) \tilde{\omega}^2 \tanh\left(\frac{\tilde{\omega}}{4}\right)$$

1. Correlator data extrapolated in the continuum

$$\tilde{G}_{ii}(\tau T) = c_{BW}\tilde{\chi}_q F_{BW}(\tau T, \tilde{\Gamma}) + (1+k(T)) \tilde{G}_V^{free}(\tau T)$$

$$F_{BW}(\tau T, \tilde{\Gamma}) = \frac{\tilde{\Gamma}}{2\pi} \int_0^\infty d\tilde{\omega} \frac{\tilde{\omega}}{(\tilde{\Gamma}/2)^2 + \tilde{\omega}^2} \frac{\cosh(\tilde{\omega}(\tau T - 1/2))}{\sinh(\tilde{\omega}/2)}$$

2. The zeroth and second thermal moments

$$\tilde{G}_{ii}^{(0)} \equiv \tilde{G}_{ii}(1/2) = c_{BW}\tilde{\chi}_q F_{BW}^{(0)}(\tilde{\Gamma}) + 2(1+k(T))$$

$$\Delta_V(\tau T) = c_{BW}\tilde{\chi}_q \frac{F_{BW}(\tau T, \tilde{\Gamma}) - F_{BW}(1/2, \tilde{\Gamma})}{\tilde{G}_V^{free}(\tau T) - \tilde{G}_V^{free}(1/2)} + 1 + k(T)$$

$$\frac{G_V^{(2)}}{G_V^{(2), free}} \equiv \Delta_V(1/2) = c_{BW}\tilde{\chi}_q \frac{F_{BW}^{(2)}(\tilde{\Gamma})}{\tilde{G}_V^{(2), free}} + 1 + k(T)$$

$$k = \alpha_s/\pi$$

$$g^2(T) = 4\pi\alpha_s \simeq 1.6$$

$$k = 0.0465(30), \tilde{\Gamma} = 2.235(75), 2c_{BW}\tilde{\chi}_q/\tilde{\Gamma} = 1.098(27)$$

good agreement with

Kaczmarek et al., PRD70(2004)074505

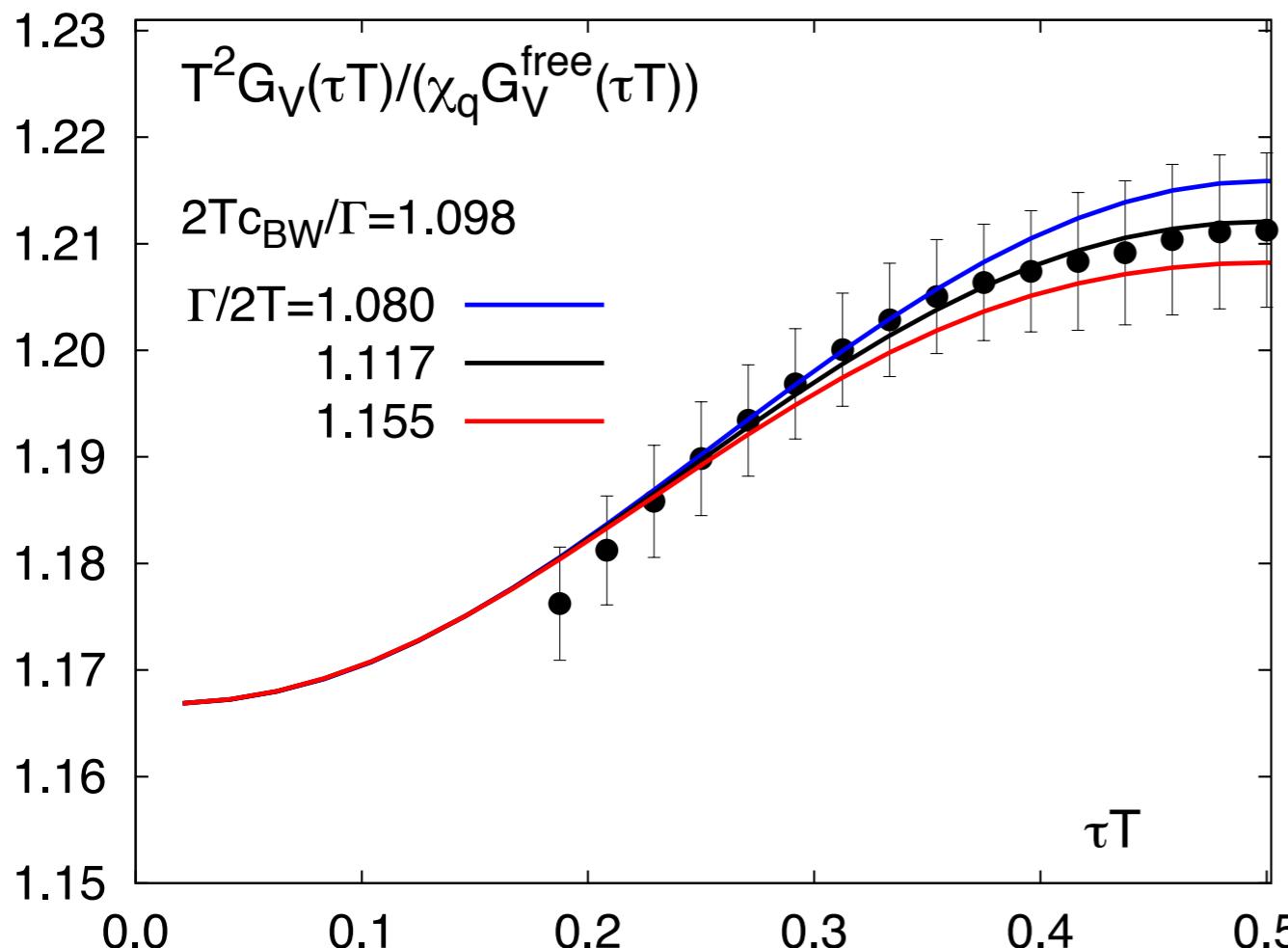
$\chi^2/d.o.f. = 0.06, d.o.f. = 12$

Dependences on the width

$$\tilde{\rho}_{ii}(\tilde{\omega}) = \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \frac{2\tilde{\omega}(\tilde{\Gamma}/2)^2}{\tilde{\omega}^2 + (\tilde{\Gamma}/2)^2} + \frac{3}{2\pi} (1+k) \tilde{\omega}^2 \tanh\left(\frac{\tilde{\omega}}{4}\right)$$

$$k = 0.0465(30), \tilde{\Gamma} = 2.235(75), 2c_{BW}\tilde{\chi}_q/\tilde{\Gamma} = 1.098(27)$$

→ vary width Γ with the other two parameters fixed

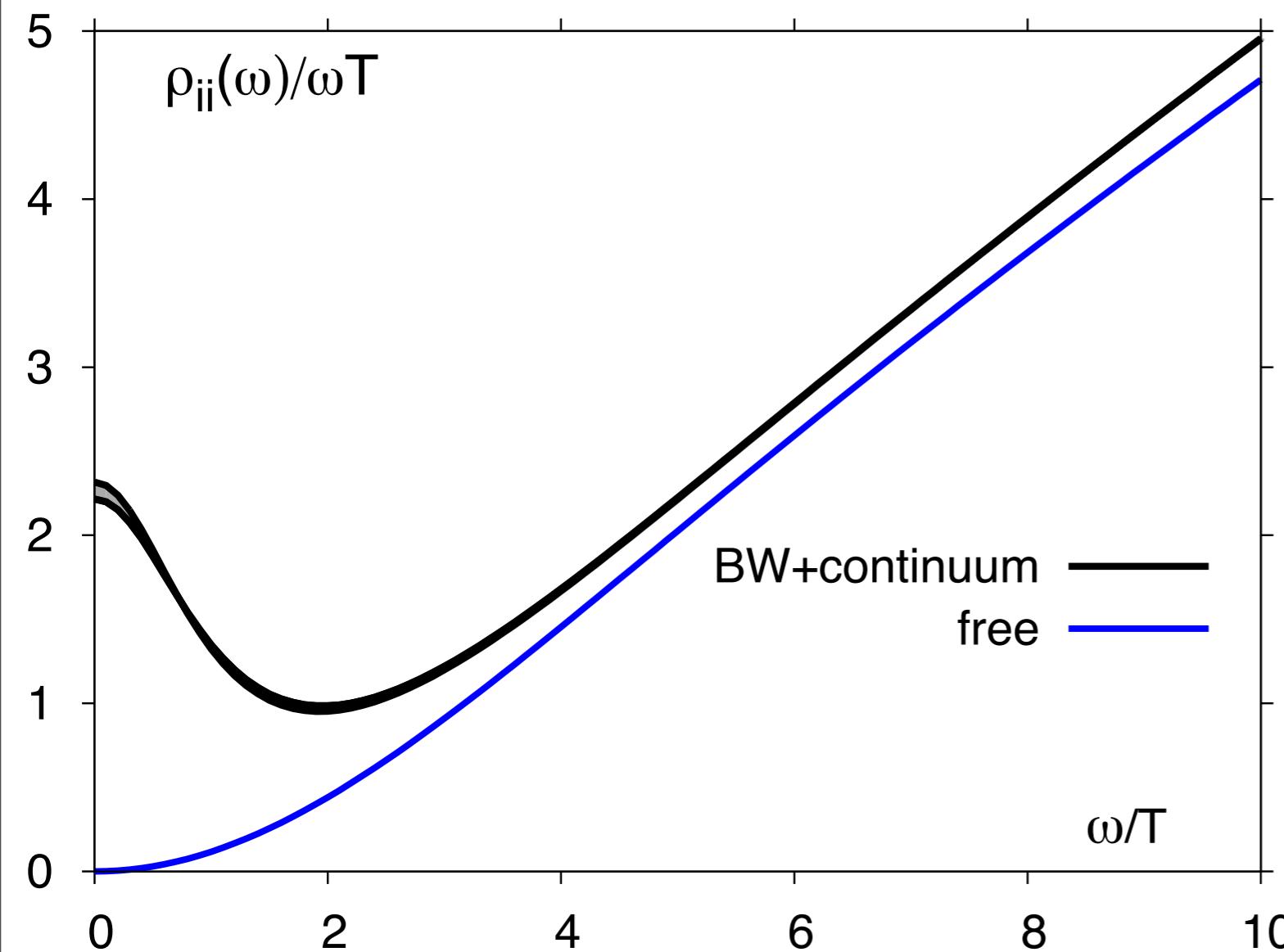


- vector correlation function is sensitive to the low energy, Breit-Wigner contribution only for distance $\tau T \gtrsim 0.25$
- fit parameters can be well constrained by the additional second thermal moment

Estimate of electrical conductivity

$$\tilde{\rho}_{ii}(\tilde{\omega}) = \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \frac{2\tilde{\omega}(\tilde{\Gamma}/2)^2}{\tilde{\omega}^2 + (\tilde{\Gamma}/2)^2} + \frac{3}{2\pi} (1+k) \tilde{\omega}^2 \tanh\left(\frac{\tilde{\omega}}{4}\right)$$

$$k = 0.0465(30), \tilde{\Gamma} = 2.235(75), 2c_{BW}\tilde{\chi}_q/\tilde{\Gamma} = 1.098(27)$$

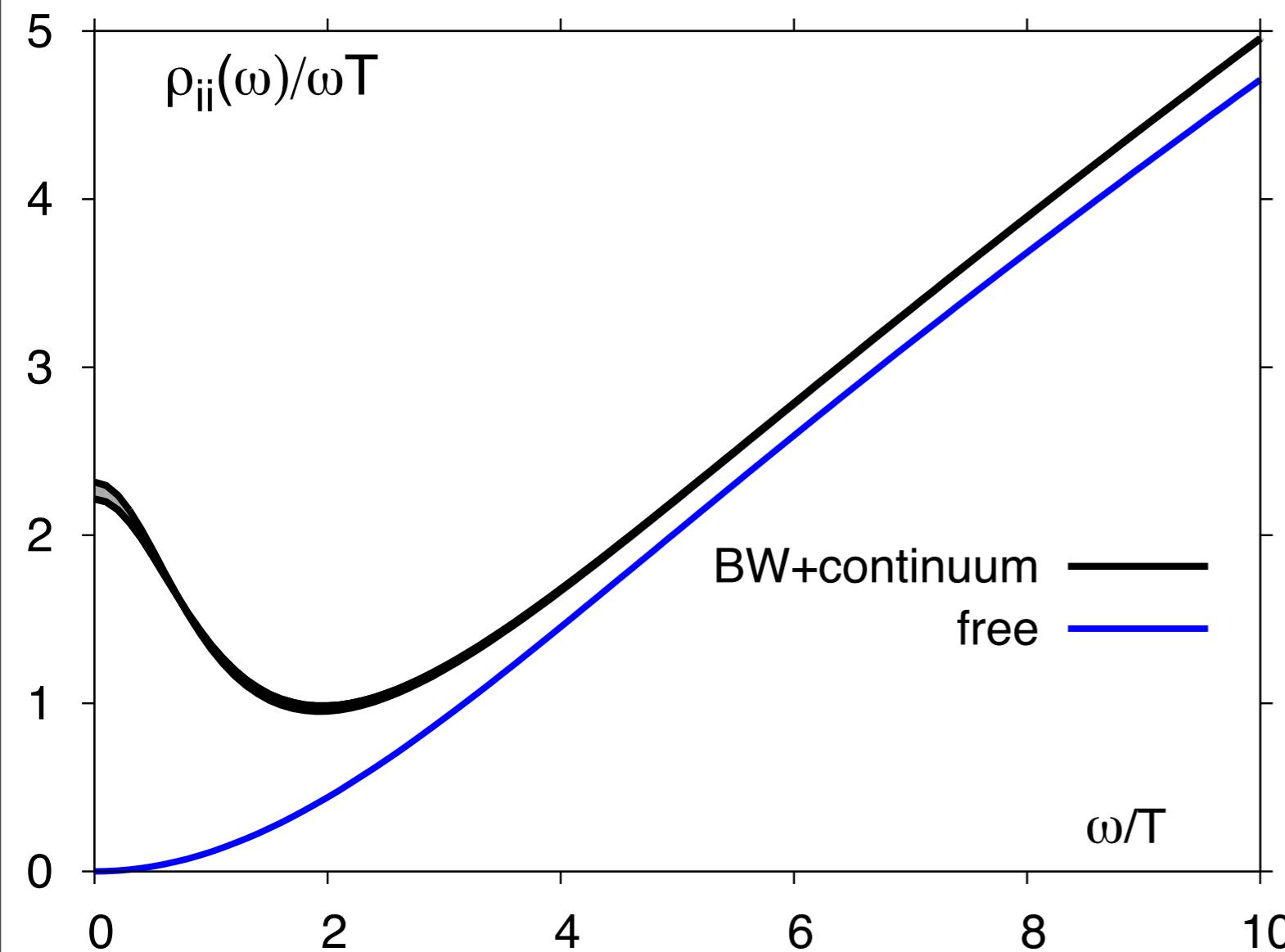


$$\begin{aligned}\frac{\sigma}{T} &= \frac{C_{em}}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega)}{\omega T} \\ &= \frac{C_{em}}{3} \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \\ &= (0.37 \pm 0.01)C_{em}\end{aligned}$$

Estimate of electrical conductivity

$$\tilde{\rho}_{ii}(\tilde{\omega}) = \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \frac{2\tilde{\omega}(\tilde{\Gamma}/2)^2}{\tilde{\omega}^2 + (\tilde{\Gamma}/2)^2} + \frac{3}{2\pi} (1+k) \tilde{\omega}^2 \tanh\left(\frac{\tilde{\omega}}{4}\right)$$

$k = 0.0465(30)$, $\tilde{\Gamma} = 2.235(75)$, $2c_{BW}\tilde{\chi}_q/\tilde{\Gamma} = 1.098(27)$



$$\begin{aligned} \frac{\sigma}{T} &= \frac{C_{em}}{6} \lim_{\omega \rightarrow 0} \frac{\rho_{ii}(\omega)}{\omega T} \\ &= \frac{C_{em}}{3} \frac{2c_{BW}\tilde{\chi}_q}{\tilde{\Gamma}} \\ &= (0.37 \pm 0.01)C_{em} \end{aligned}$$

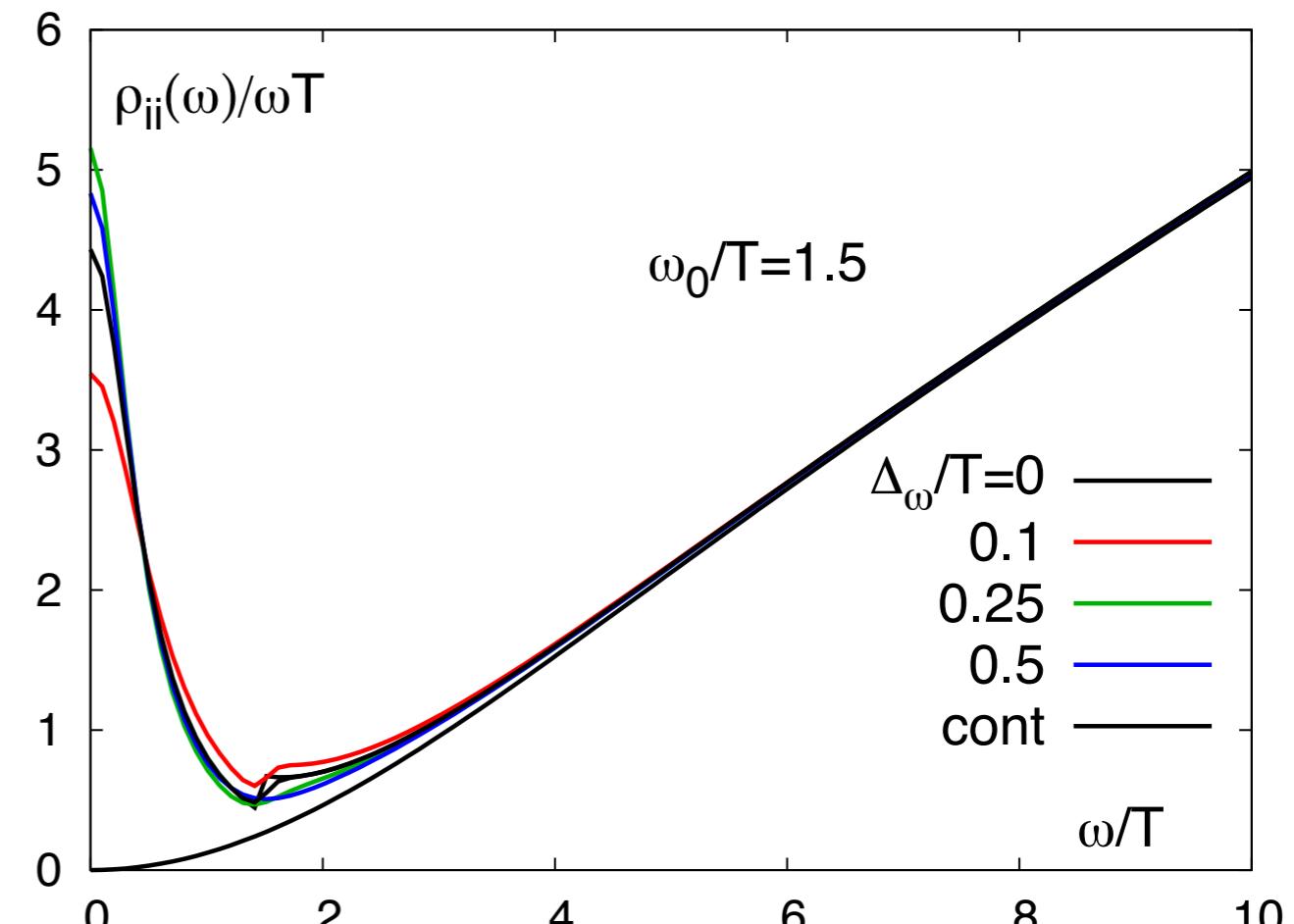
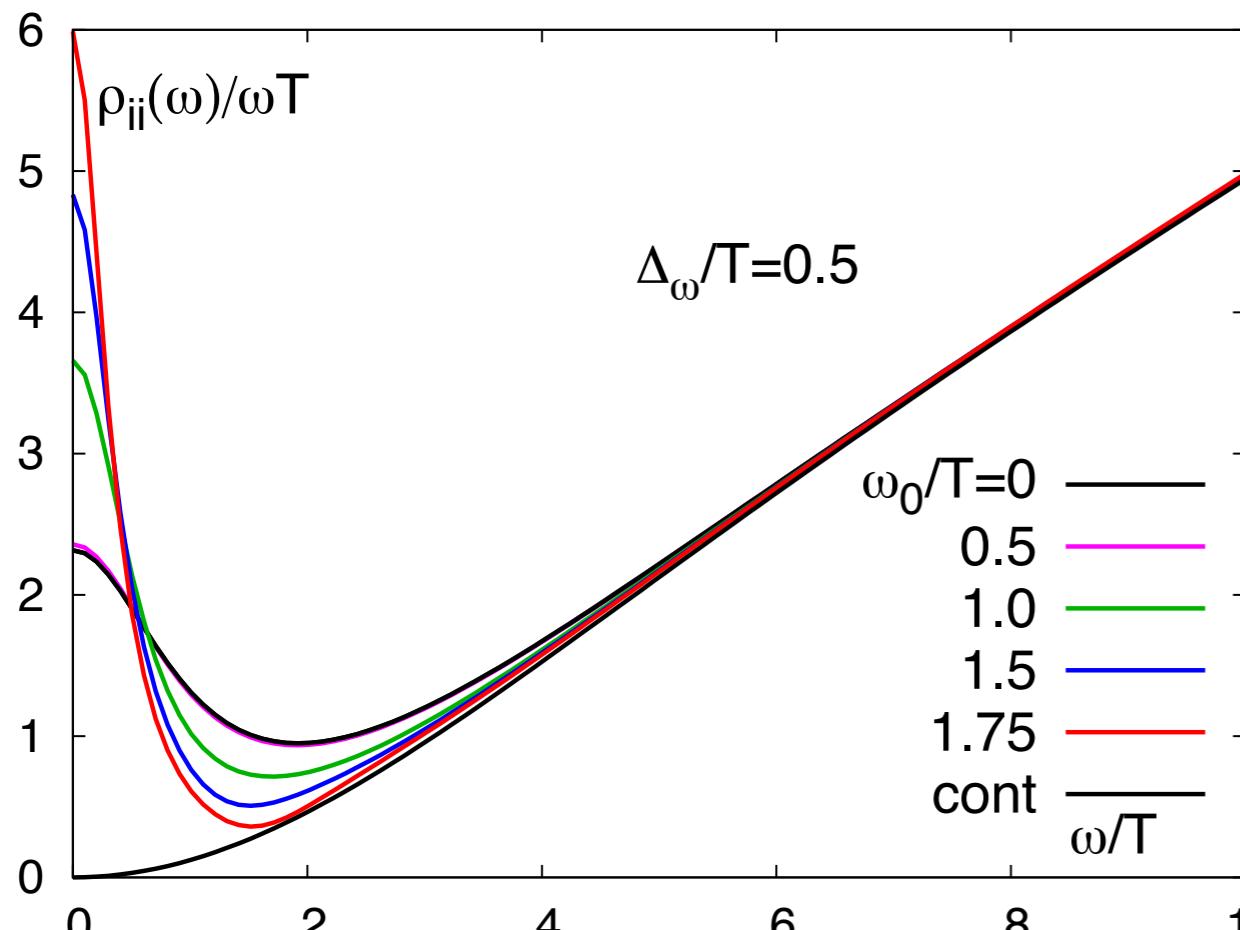
(accidentally) close to
Aarts' result!

Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1+k) \omega^2 \tanh\left(\frac{\omega}{4T}\right) \Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

delay the onset (ω_0) of the continuum part



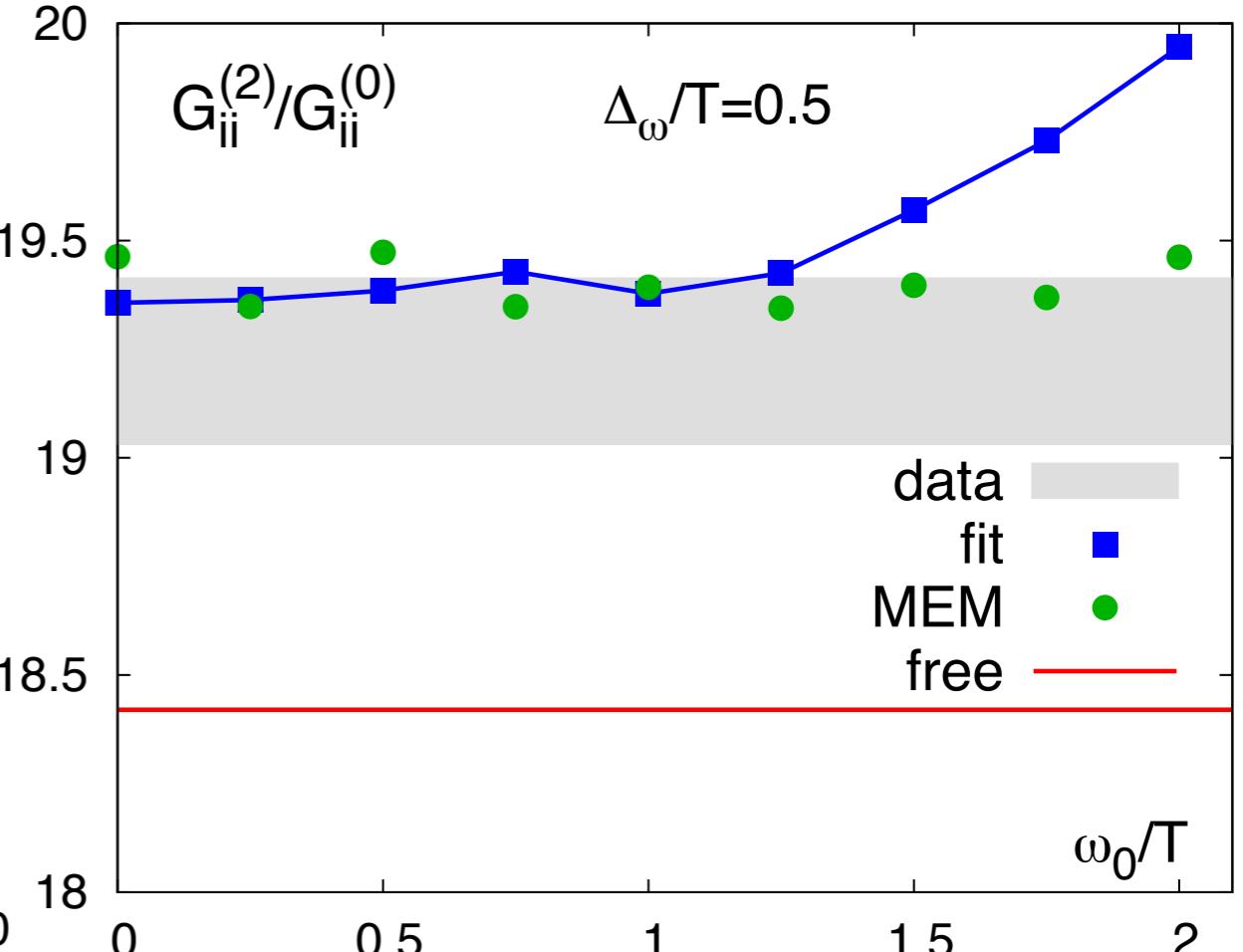
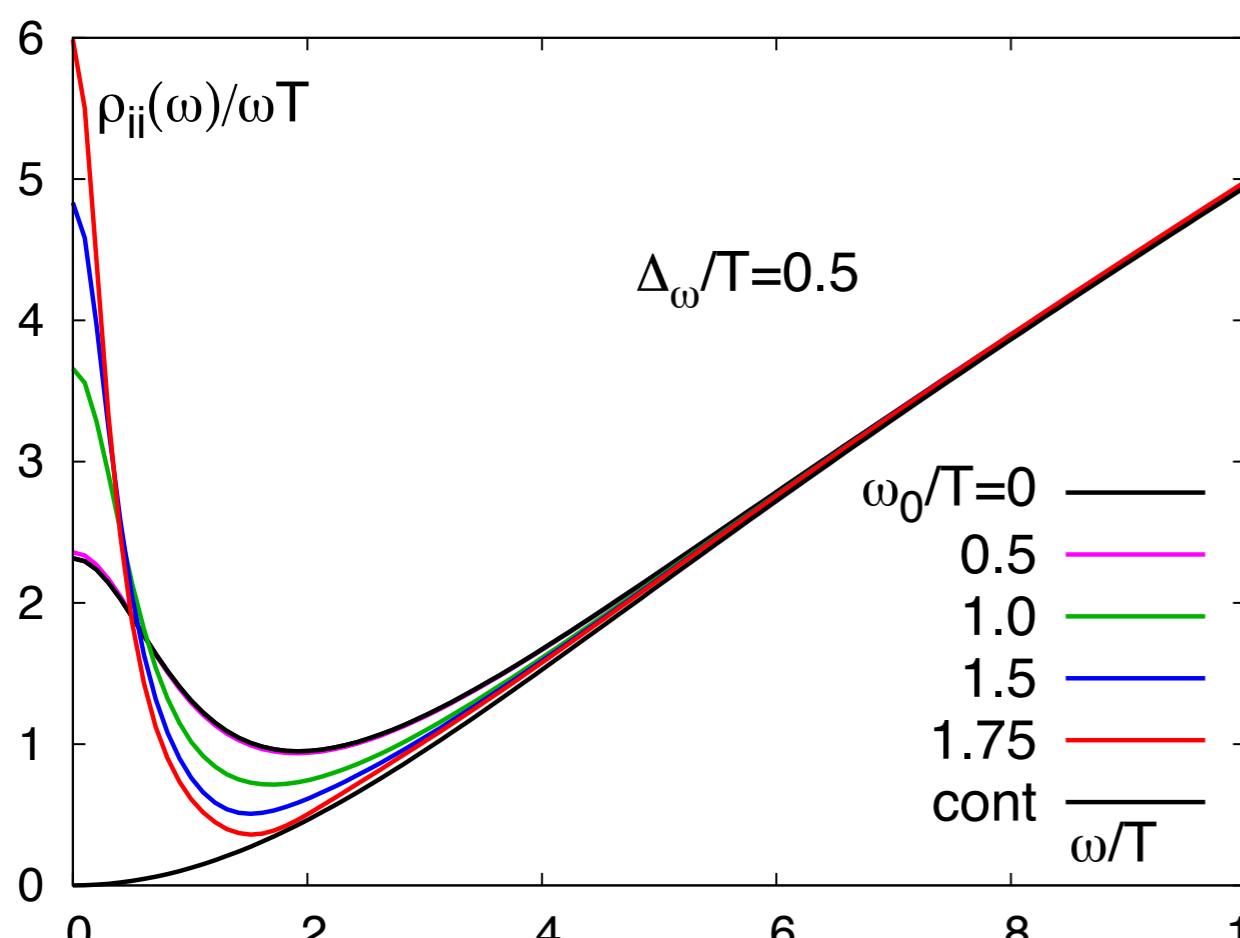
- Rise of BW peaks compensate for the cut from continuum parts
- Fits become worse with increasing ω_0 and/or increasing Δ_ω

Breit-Wigner + truncated continuum Ansatz

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1+k) \omega^2 \tanh\left(\frac{\omega}{4T}\right) \Theta(\omega_0, \Delta_\omega)$$

$$\Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

delay the onset (ω_0) of the continuum part



- Ratio of thermal moments reacts sensitive to the truncation of the cont. part
- Spectral function should not deviate much from free behavior for $\omega/T \gtrsim (2 - 4)$

Maximum Entropy Method

[Asakawa, Nakahara & Hatsuda '96]

- Hard to extract spectral function (spf)

$$G(\tau, T) = \int_0^\infty d\omega K(\tau, \omega, T) \sigma(\omega, T)$$

Discretized
 $\mathcal{O}(10)$ Continuous
 $\mathcal{O}(10^3)$

- Maximum Entropy Method (MEM), based on Bayesian theorem

- A method to obtain the most probable image from insufficient data
- Ingredients of MEM: $P[\sigma|GH] \propto P[G|\sigma H] P[\sigma|H]$

$P[G|\sigma H] \propto \exp(-\chi^2/2)$: likelihood function

$P[\sigma|H] \propto \exp(\alpha S)$: prior probability

σ : spectral function
G: lattice data
H: prior information on σ

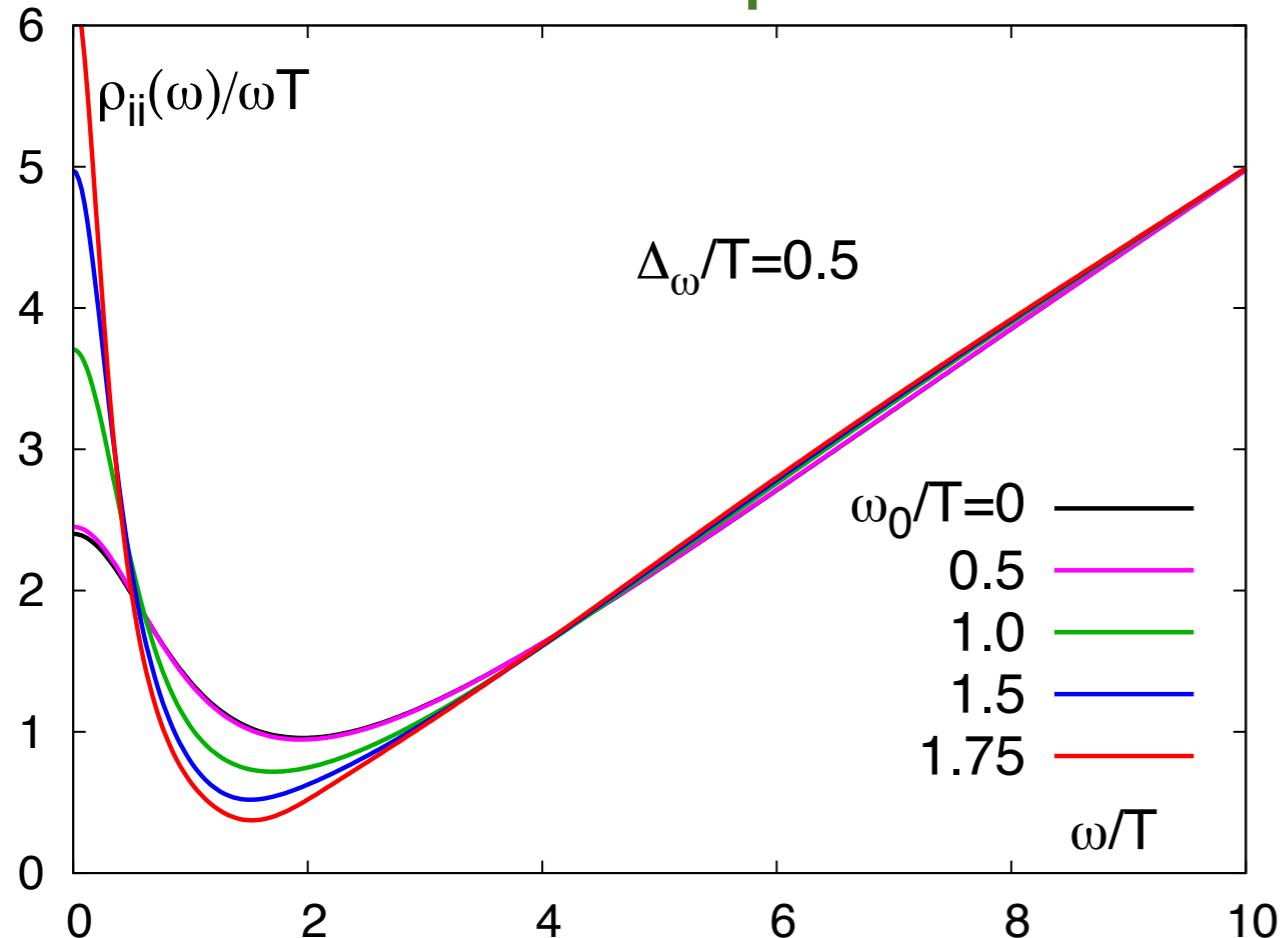
Information entropy: $S = \int_0^\infty d\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln \left(\frac{\sigma(\omega)}{m(\omega)} \right) \right]$

Default Model (DM): $m(\omega)$, includes the prior information on σ , e.g. σ is positive-definite
DM is the **only** input parameter in the MEM analysis

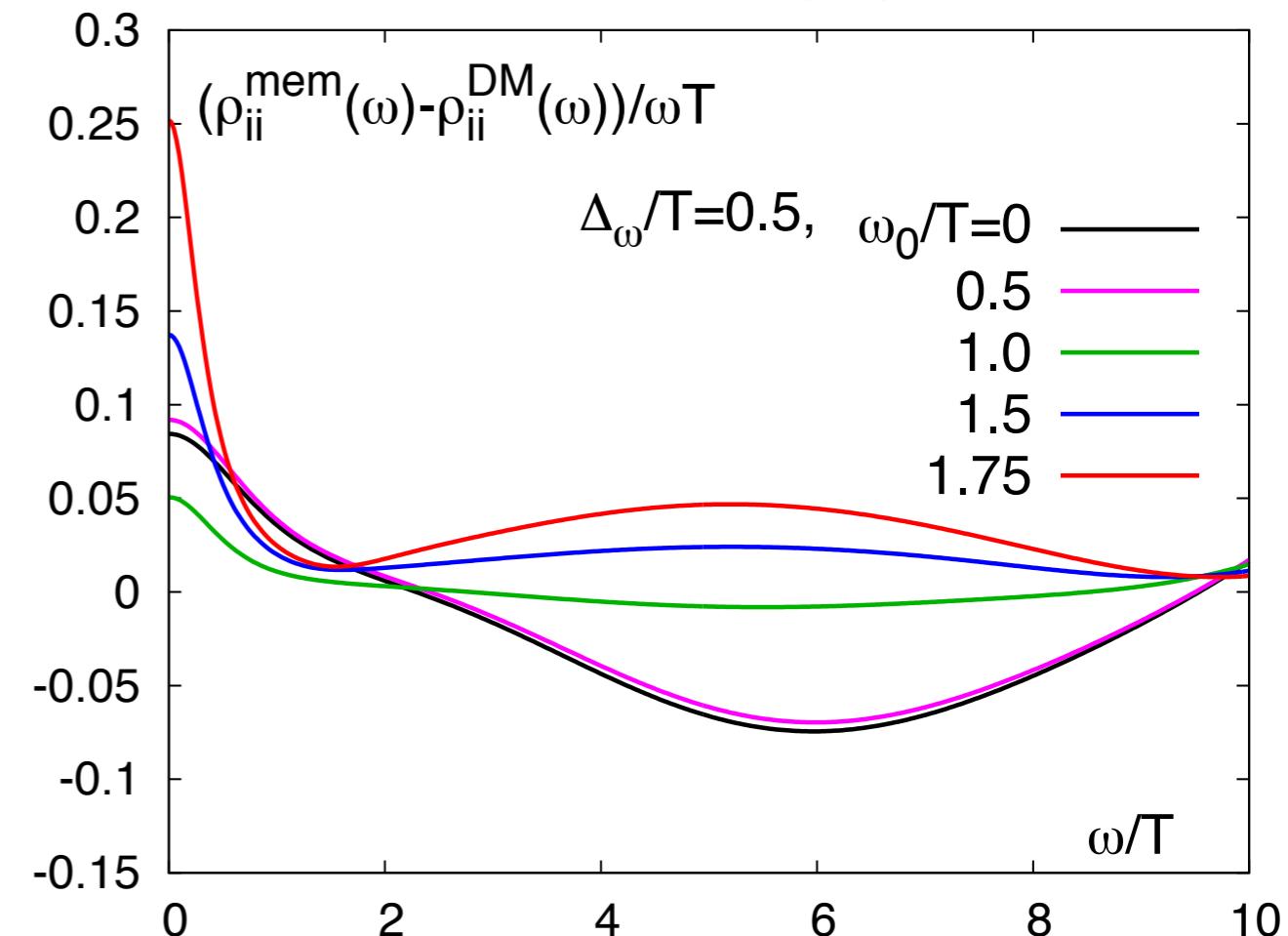
- Maximizing $P[\sigma|GH]$ gives the most probable spectral function
- Important to check the dependence of output spfs on DMs

MEM analysis

MEM outputs



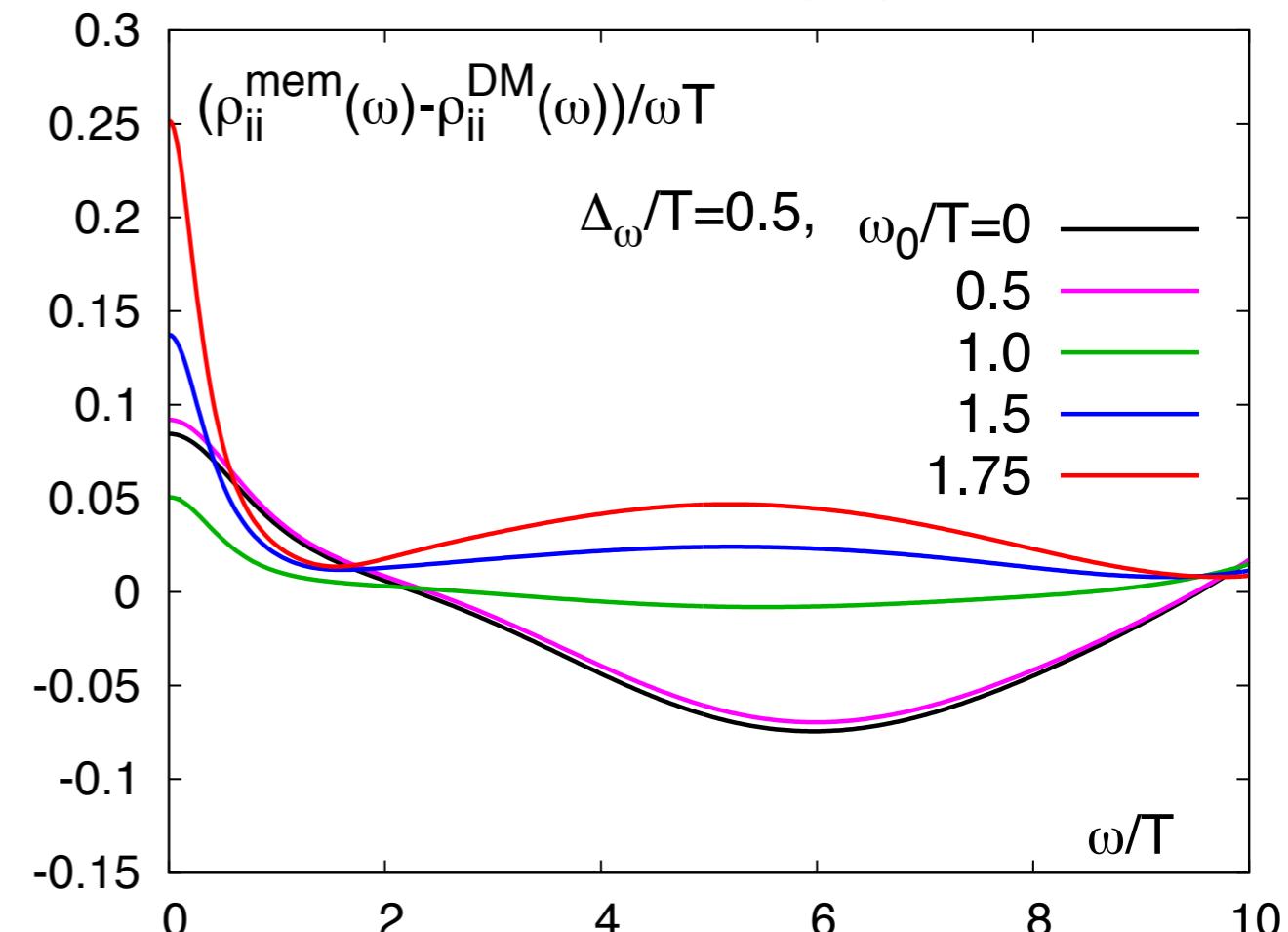
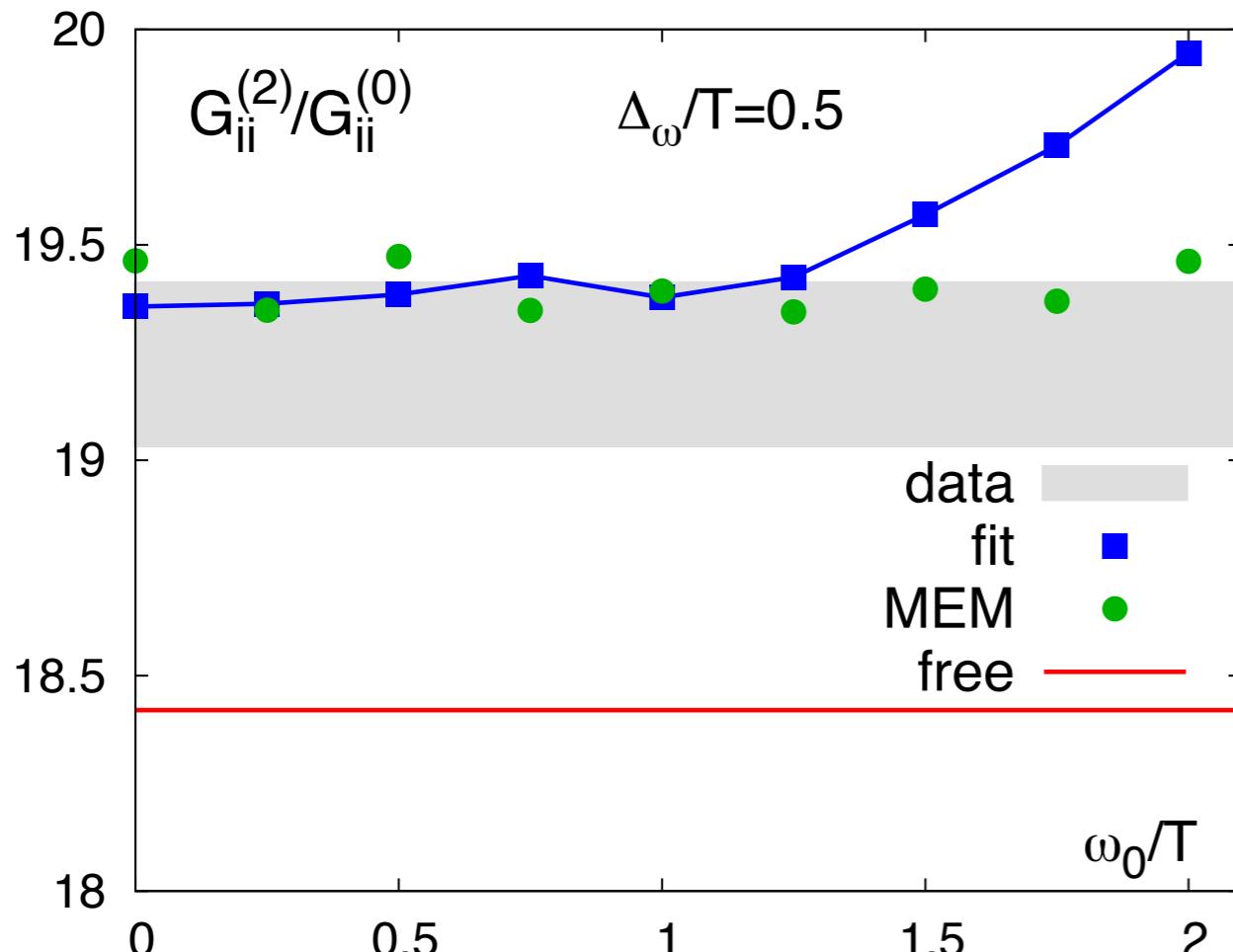
Differences



- Differences are smaller than 2% with smaller parameters and increase with worse default models

MEM analysis

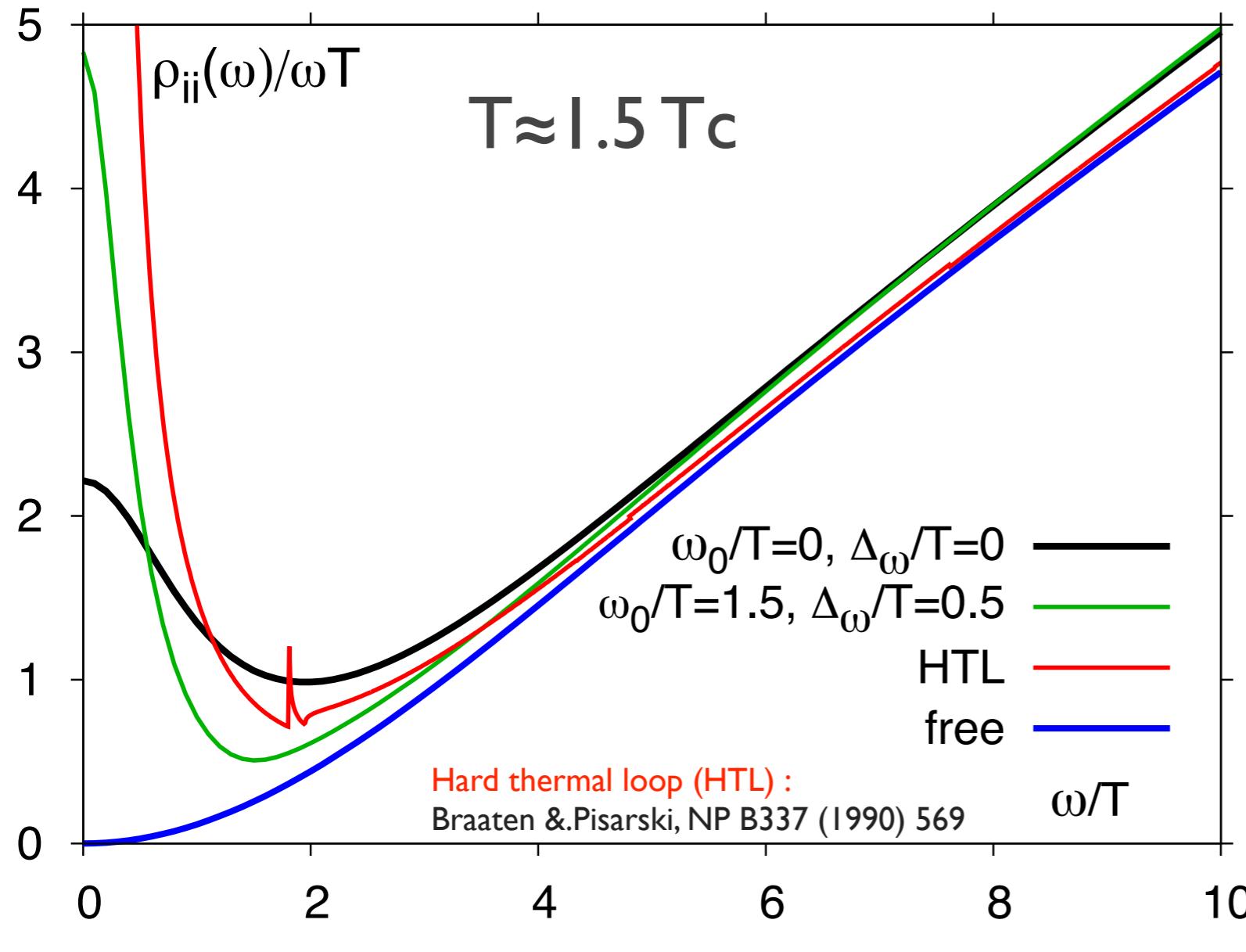
Differences



- Differences are smaller than 2% with smaller parameters and increase with worse default models
- MEM analysis reproduces the ratio of thermal moments even better
- Statistical errors are small

Electrical conductivity

$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega\Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi} (1+k) \omega^2 \tanh\left(\frac{\omega}{4T}\right) \Theta(\omega_0, \Delta_\omega)$$



$$\Theta(\omega_0, \Delta_\omega) = \left(1 + e^{(\omega_0^2 - \omega^2)/\omega\Delta_\omega}\right)^{-1}$$

electrical conductivity

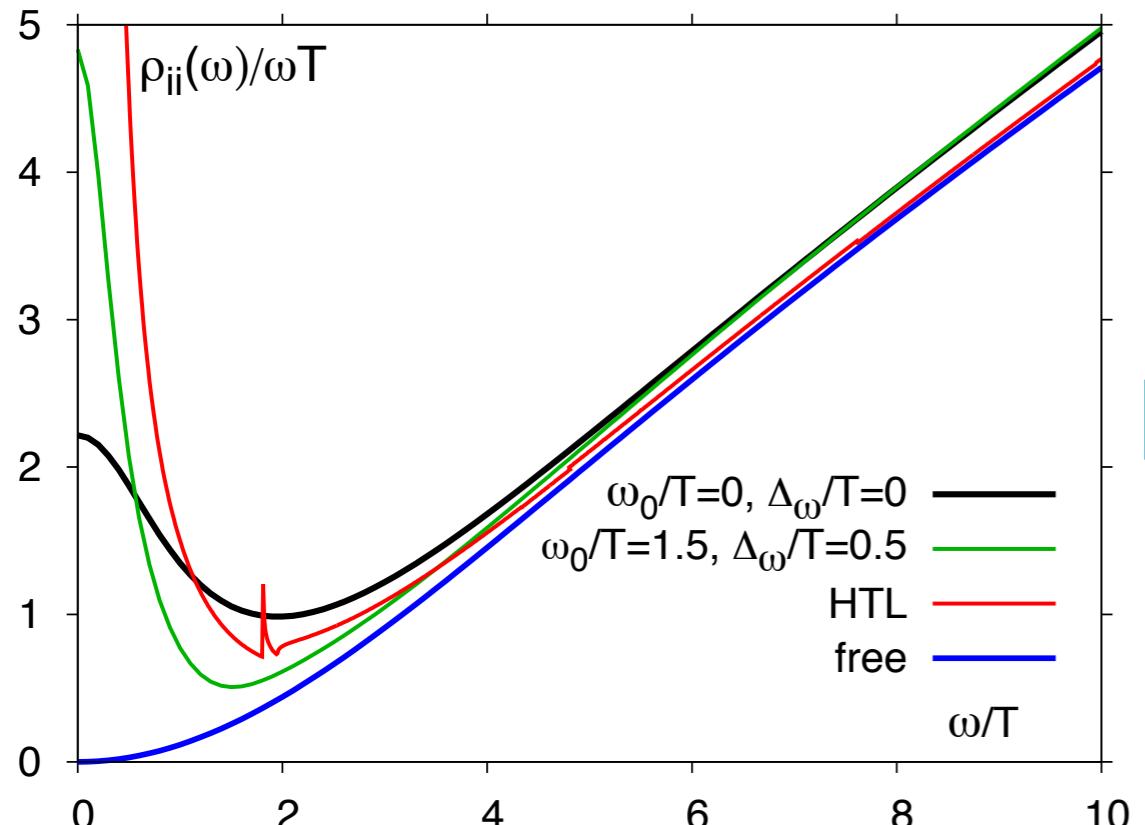
$$1/3 \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 1$$

Soft photon emission rate

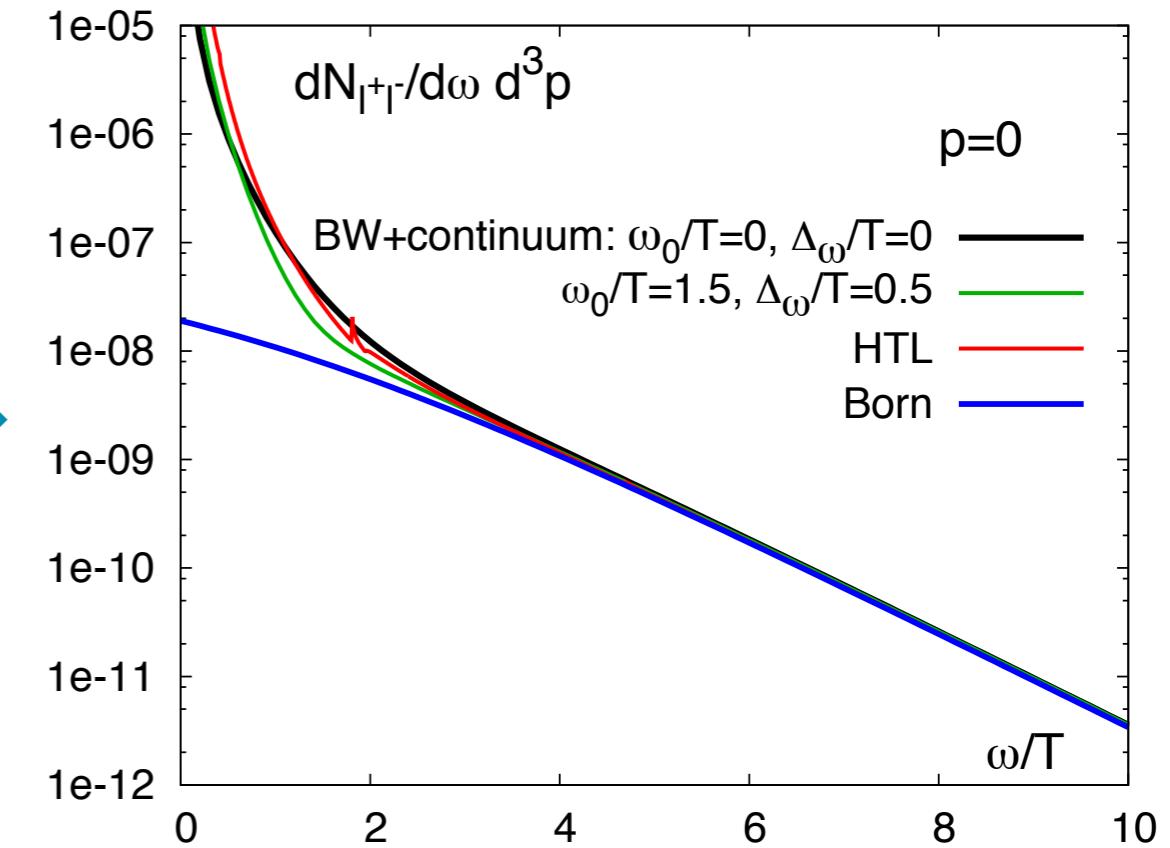
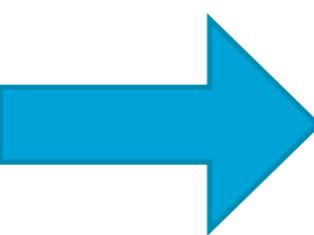
$$\lim_{\omega \rightarrow 0} \omega \frac{dR_\gamma}{d^3 p} = (0.0004 - 0.0013) T_c^2$$

Thermal dilepton rates

$$\frac{dN_{l^+l^-}}{d\omega d^3p} = C_{em} \frac{\alpha_{em}^2}{6\pi^3} \frac{\rho_V(\omega, \vec{p}, T)}{(\omega^2 - \vec{p}^2)(e^{\omega/T} - 1)}$$



Hard thermal loop (HTL): Braaten & Pisarski, NP B337 (1990) 569



HTD, Francis, Kaczmarek, Karsch, Laermann, Soeldner, Phys. Rev. D83 (2011) 034504

- thermal dilepton rate approaches leading order Born rate at $w/T \gtrsim 4$
- enhancement at small w/T

Conclusions & Outlook

- We calculated the vector correlation function at $T \approx 1.45 T_c$ in quenched lattice QCD and performed a continuum extrapolation
- $G_v(\tau T)$ is well reproduced using a Breit-Wigner plus continuum ansatz for the vector spectral function
- Electrical conductivity $1/3 \lesssim \frac{1}{C_{em}} \frac{\sigma}{T} \lesssim 1$ at $T \simeq 1.45 T_c$
- Dilepton rate approaches leading order Born rate at $\omega/T \gtrsim 4$
 - ▶ Vector correlation functions at other temperatures and non-zero momenta